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Research Paper

Common Fixed Point Theorem Using Compatible Mapping of Type (α) in Intuitionistic Fuzzy Metric Spaces

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Abstract: *The aim of this paper is to prove a common fixed theorem under the condition of compatible mapping of type (α) in intuitionistic fuzzy metric space.*

Keywords: Intuitionistic fuzzy metric space, common fixed point, compatible mapping, compatible mapping of type (α).

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1. Introduction:

Zadeh [15] was first investigated the theory of fuzzy sets in 1965. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [8] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et. al.[1] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous *t*-norm and continuous *t*-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [7]. Park [8] introduced and discussed a notion of intuitionistic fuzzy metric spaces (briefly, IFM-spaces), which is based both on the idea of intuitionistic fuzzy sets and the concept of a fuzzy metric space given by George and Veeramani [5]. Turkoglu et al. [14] introduced the concept of compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces. They also gave some relations between the concepts of compatible mapping and compatible mappings of type (α) and (β). In 2008, Altun and Turkoglu [2] proved two common fixed point theorems for continuous compatible maps of type (α) and (β) on a complete fuzzy metric space with an implicit relation.

In 2009, Cho, Sedghi and Shobe [4] introduced definitions of compatible mappings of type (I) and (II) in fuzzy metric space. In 2010, Park [9] proved theorem for semi-compatible maps which extended and generalized the result of Singh and Chouhan [13] and in same year he gave definitions of compatible mappings of type (γ) in intuitionistic fuzzy metric space and obtained fixed point theorems under the conditions of weak compatible mappings of type (γ) in complete intuitionistic fuzzy metric space, which generalizes, extends and improves the results given by Sedghi et al. [12]. In this paper we prove a common fixed theorem under the condition of compatible mapping of type (α) in intuitionistic fuzzy metric spaces.

2. Preliminaries:

Definition (2.1) [8]: *A binary operation* $*: [0, 1] \times [0, 1] \to [0, 1]$ *is continuous t-norm if* $*$ *is satisfying the following conditions:*

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0, 1]$;
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0, 1]$.

Definition (2.2) [8]: *A binary operation* $\lozenge: [0, 1] \times [0, 1] \to [0, 1]$ *is continuous t-conorm if* $\lozenge$ *is satisfying the following conditions:*

(i) $\lozenge$ is commutative and associative;
(ii) $\lozenge$ is continuous;
(iii) $a \lozenge 0 = a$ for all $a \in [0, 1]$;
(iv) $a \lozenge b \le c \lozenge d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0, 1]$.

Definition (2.3) [1]: *A 5-tuple* $(X, M, N, *, \lozenge)$ *is said to be an intuitionistic fuzzy metric space if X is an arbitrary set,* $*$ *is a continuous t-norm,* $\lozenge$ *is a continuous t-conorm and M, N are fuzzy sets on* $X^2 \times (0, \infty)$ *satisfying the following conditions:*

(i) $M(x, y, t) + N(x, y, t) \le 1$ for all $x, y \in X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;
(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(v) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(vi) For all $x, y \in X$, $M(x, y, \cdot): [0, \infty) \to [0, 1]$ is continuous;
(vii) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;
(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;
(ix) $N(x, y, t) = 0$ for all $x, y \in X$ *and* $t > 0$ if and only if $x = y$;
(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x, y, t) \lozenge N(y, z, s) \ge N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(xii) For all $x, y \in X$, $N(x, y, \cdot) : [0, \infty) \to [0, 1]$ is continuous;
(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all x, y in X.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark (2.1): Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \lozenge)$ such that t-norm $*$ and t-conorm $\lozenge$ are associated as

$x \lozenge y = 1- ((1-x) * (1-y))$ for all $x, y \in X$.

Remark (2.2): In intuitionistic fuzzy metric space X, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all $x, y \in X$.

Example (2.1) [7]: Let (X, d) be a metric space, define t-norm $a * b = \min\{a, b\}$ and t-conorm $a \lozenge b = \max\{a, b\}$ and for all $x, y \in X$ and $t > 0$,

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \; N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Then $(X, M, N, *, \lozenge)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric.

Definition (2.4) [1]: *Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space. Then*

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1, \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$$\lim_{n\to\infty} M(x_n, x, t) = 1, \lim_{n\to\infty} N(x_n, x, t) = 0.$$

Since $*$ and $\lozenge$ are continuous, the limit is uniquely determined from (v) and (xi) of definition (2.3), respectively.

Definition (2.5) [1]: *An intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ is said to be complete if and only if every Cauchy sequence in X is convergent.*

Definition (2.6) [10]: *Let A and B be mappings from an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ into itself. The maps A and B are said to be compatible if, for all $t > 0$,*

$$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0,$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Definition (2.7) [6]: *Two self maps A and B of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ are said to be weak compatible if they commute at their coincidence points.*

Definition (2.8) [14]: *Let A and B be maps from an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ into itself. The maps A and B are said to be compatible of type (α) if, for all $t > 0$,*

$$\lim_{n\to\infty} M(ABx_n, BBx_n, t) = 1 \text{ and } \lim_{n\to\infty} M(ABx_n, BBx_n, t) = 0,$$

$$\lim_{n\to\infty} M(BAx_n, AAx_n, t) = 1 \text{ and } \lim_{n\to\infty} M(BAx_n, AAx_n, t) = 0,$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Remark (2.3): If self maps A and B of a intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ are compatible of type (α), then they are weak compatible.

Lemma (2.1) [1]: *Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. if there exists a number $k \in (0, 1)$, such that*

$$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t) \text{ and } N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t),$$

for all $t > 0$ *and* $n = 1, 2, \ldots$, *then* $\{y_n\}$ *is a Cauchy sequence in X.*

Lemma (2.2) [14]: *Let* $(X, M, N, *, \Diamond)$ *be an intuitionistic fuzzy metric space and for all x, y in X,* $t > 0$ *and if there exists a number* $k \in (0, 1)$

$M(x, y, kt) \geq M(x, y, t)$ *and* $N(x, y, kt) \leq N(x, y, t)$, *then* $x = y$.

3. Main Result:

In this section, we establish a common fixed point theorem in intuitionistic fuzzy metric space under the condition of compatible mappings of type (α).

Theorem (3.1): *Let* $(X, M, N, *, \Diamond)$ *be a intuitionistic fuzzy metric space and let A, B, S and T be self maps from X such that*

(3.1) $AT(X) \cup BS(X) \subseteq ST(X)$
(3.2) the pairs (A, S) and (B, T) are compatible maps of type (α);
(3.3) $AB = BA$, $ST = TS$;
(3.4) one maps from each of the above two pairs of (3.2) are continuous;
(3.5) there exists a constant $k \in (0, 1)$ such that

$[1 + aM(Sx, Ty, kt)]*M(Ax, By, kt) \geq a\text{Min}\{M(By, Ty, kt), M(Ax, Sx, kt), M(Ax, Ty, 2kt),$

$M(By, Sx, 2kt)\} + \text{Min}\{M(Ax, Sx, t), M(Sx, Ty, t), M(By, Ty, t), M(Ax, Ty, 2t), M(By, Sx, 2t)\}$

and $[1 + aN(Sx, Ty, kt)]\Diamond N(Ax, By, kt) \leq a\text{Max}\{N(By, Ty, kt), N(Ax, Sx, kt), N(Ax, Ty, 2kt),$

$N(By, Sx, 2kt)\} + \text{Max}\{N(Ax, Sx, t), N(Sx, Ty, t), N(By, Ty, t), N(Ax, Ty, 2t), N(By, Sx, 2t)\}$

for all $x, y \in X$ and $t > 0$ with fixed constant $a \in (-1, 0)$.

Then A, B, S and T have a unique common fixed point in X.

Proof: Let x_0 be an arbitrary point X. Now, by (3.1) we can construct a sequence $\{y_n\}$ in X such that,

$$y_{2n+1} = ATx_{2n} = STx_{2n+1} \text{ and } y_{2n+1} = BSx_{2n+1} = STx_{2n+2} \tag{3.6}$$

Step-(1): First we show that the sequence $\{y_n\}$ converges to any point of X.

Using the condition (3.3) and taking $x = Tx_{2n}$ and $y = Sx_{2n+1}$ in (3.5), we get

$[1 + aM(STx_{2n}, TSx_{2n+1}, kt)]*M(ATx_{2n}, BSx_{2n+1}, kt) \geq a\text{Min}\{M(BSx_{2n+1}, TSx_{2n+1}, kt),$

$M(ATx_{2n}, STx_{2n}, kt), M(ATx_{2n}, TSx_{2n+1}, 2kt), M(BSx_{2n+1}, STx_{2n}, 2kt)\}+\text{Min}\{M(ATx_{2n}, STx_{2n}, t),$

$M(STx_{2n}, TSx_{2n+1}, t), M(BSx_{2n+1}, TSx_{2n+1}, t), M(ATx_{2n}, TSx_{2n+1}, 2t), M(BSx_{2n+1}, STx_{2n}, 2t)\}$

and $[1 + aN(STx_{2n}, TSx_{2n+1}, kt)]\Diamond N(ATx_{2n}, BSx_{2n+1}, kt) \leq a\text{Max}\{N(BSx_{2n+1}, TSx_{2n+1}, kt),$

$N(ATx_{2n}, STx_{2n}, kt), N(ATx_{2n}, TSx_{2n+1}, 2kt), N(BSx_{2n+1}, STx_{2n}, 2kt)\}+\text{Max}\{N(ATx_{2n}, STx_{2n}, t),$

$N(STx_{2n}, TSx_{2n+1}, t), N(BSx_{2n+1}, TSx_{2n+1}, t), N(ATx_{2n}, TSx_{2n+1}, 2t), N(BSx_{2n+1}, STx_{2n}, 2t)\}.$

Now using (3.6) and (3.3), we get,

$[1 + aM(y_{2n}, y_{2n+1}, kt)]*M(y_{2n+1}, y_{2n+2}, kt) \geq a\text{Min}\{M(y_{2n+2}, y_{2n+1}, kt), M(y_{2n+1}, y_{2n}, kt),$

$M(y_{2n+1}, y_{2n}, 2kt), M(y_{2n+2}, y_{2n}, 2kt)\}+\text{Min}\{M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n+2}, y_{2n+1}, t),$

$M(y_{2n+1}, y_{2n+1}, 2t), M(y_{2n+2}, y_{2n}, 2t)\}$

$M(y_{2n+1}, y_{2n+2}, kt)+aM(y_{2n}, y_{2n+1}, kt)*M(y_{2n+1}, y_{2n+2}, kt) \geq a\text{Min}\{M(y_{2n+2}, y_{2n+1}, kt),$

$M(y_{2n+2}, y_{2n}, 2kt)\}+\text{Min}\{M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+2}, y_{2n}, 2t)\}$

$M(y_{2n+1}, y_{2n+2}, kt) + aM(y_{2n}, y_{2n+2}, kt) \geq aM(y_{2n}, y_{2n+2}, kt) + M(y_{2n}, y_{2n+1}, t)$

and $[1 + aN(y_{2n}, y_{2n+1}, kt)]\Diamond N(y_{2n+1}, y_{2n+2}, kt) \leq a\text{Max}\{N(y_{2n+2}, y_{2n+1}, kt), N(y_{2n+1}, y_{2n}, kt),$

$N(y_{2n+1}, y_{2n}, 2kt), N(y_{2n+2}, y_{2n}, 2kt)\}+\text{Max}\{N(y_{2n+1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n+2}, y_{2n+1}, t)$

$N(y_{2n+1}, y_{2n+1}, 2t), N(y_{2n+2}, y_{2n}, 2t)\}$

$N(y_{2n+1}, y_{2n+2}, kt)+aN(y_{2n}, y_{2n+1}, kt)\Diamond N(y_{2n+1}, y_{2n+2}, kt) \leq a\text{Max}\{N(y_{2n+2}, y_{2n+1}, kt),$

$N(y_{2n+2}, y_{2n}, 2kt)\}+\text{Max}\{N(y_{2n+2}, y_{2n+1}, t), N(y_{2n+2}, y_{2n}, 2t)\}$

$N(y_{2n+1}, y_{2n+2}, kt) + aN(y_{2n}, y_{2n+2}, kt) \leq aN(y_{2n}, y_{2n+2}, kt) + N(y_{2n}, y_{2n+1}, t)$

That is, $M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$ and $N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t)$.

For all $k \in (0, 1)$ and $t > 0$. Similarly, we also have $M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t)$

and $N(y_{2n+2}, y_{2n+3}, kt) \leq N(y_{2n+1}, y_{2n+2}, t)$, for all $k \in (0, 1)$ and $t > 0$.

In general, for $m = 1, 2, 3, \ldots$, we have $M(y_{m+1}, y_{m+2}, kt) \geq M(y_m, y_{m+1}, t)$

and $N(y_{m+1}, y_{m+2}, kt) \leq N(y_m, y_{m+1}, t)$, for all $k \in (0, 1)$ and $t > 0$.

Therefore by lemma (2.1), $\{y_n\}$ is a Cauchy sequence in X, which is complete.

Hence $\{y_n\} \to v \in X$, and its subsequences $ATx_{2n} \to v$ and $STx_{2n} \to v$, $BSx_{2n+1} \to v$ and $STx_{2n+1} \to v$.

Let $Tx_{2n} = u_n$ and $Sx_{2n+1} = w_{n+1}$ for all $n > 0$, then $Au_n = v$ and $Su_n = v$ (3.7)

$Bw_{n+1} = v$ and $Tw_{n+1} = v$. (3.8)

Case-(I): A and B are continuous. As A is continuous we have $A^2u_n = Av$. (3.9)

As (A, S) is compatible of type (α), then by (3.7) we get $SAu_n \to Av$. (3.10)

Step-(2): Taking $x = Au_n$, $y = w_{n+1}$ in (3.5) we get,

$[1 + aM(SAu_n, Tw_{n+1}, kt)]*M(AAu_n, Bw_{n+1}, kt) \geq a\text{Min}\{M(Bw_{n+1}, Tw_{n+1}, kt),$

$M(AAu_n, SAu_n, kt), M(AAu_n, Tw_{n+1}, 2kt), M(Bw_{n+1}, SAu_n, 2kt)\}+\text{Min}\{M(AAu_n, SAu_n, t),$

$M(SAu_n, Tw_{n+1}, t), M(Bw_{n+1}, Tw_{n+1}, t), M(AAu_n, Tw_{n+1}, 2t), M(Bw_{n+1}, SAu_n, 2t)\}$

$[1 + aM(Av, v, kt)]*M(Av, v, kt) \geq a\text{Min}\{M(v, v, kt), M(Av, Av, kt), M(Av, v, 2kt),$

$M(v, Av, 2kt)\}+\text{Min}\{M(Av, Av, t), M(Av, v, t), M(v, v, t), M(Av, v, 2t), M(v, Av, 2t)\}$

$M(Av, v, kt) + aM(Av, v, kt)*M(Av, v, kt) \geq a\text{Min}\{1, 1, M(Av, v, 2kt), M(v, Av, 2kt)\}$

$+\text{Min}\{1, M(Av, v, t), 1, M(Av, v, 2t), M(v, Av, 2t)\}$

$M(Av, v, kt) + aM(Av, v, kt) \geq aM(Av, v, kt) + M(Av, v, t)$

and $[1 + aN(SAu_n, Tw_{n+1}, kt)]\Diamond N(AAu_n, Bw_{n+1}, kt) \leq a\text{Max}\{N(Bw_{n+1}, Tw_{n+1}, kt),$

$N(AAu_n, SAu_n, kt), N(AAu_n, Tw_{n+1}, 2kt), N(Bw_{n+1}, SAu_n, 2kt)\}+\text{Max}\{N(AAu_n, SAu_n, t),$

$N(SAu_n, Tw_{n+1}, t), N(Bw_{n+1}, Tw_{n+1}, t), N(AAu_n, Tw_{n+1}, 2t), N(Bw_{n+1}, SAu_n, 2t)\}$

$[1 + aN(Av, v, kt)]\Diamond N(Av, v, kt) \leq a\text{Max}\{N(v, v, kt), N(Av, Av, kt), N(Av, v, 2kt),$

$N(v, Av, 2kt)\}+\text{Max}\{N(Av, Av, t), N(Av, v, t), N(v, v, t), N(Av, v, 2t), N(v, Av, 2t)\}$

$N(Av, v, kt) + aN(Av, v, kt)\Diamond N(Av, v, kt) \leq a\text{Max}\{0, 0, N(Av, v, 2kt), N(v, Av, 2kt)\}$

$+\text{Max}\{0, N(Av, v, t), 0, N(Av, v, 2t), N(v, Av, 2t)\}$

$N(Av, v, kt) + aN(Av, v, kt) \leq aN(Av, v, kt) + N(Av, v, t)$

That is, $M(Av, v, kt) \geq M(Av, v, t)$ and $N(Av, v, kt) \leq N(Av, v, t)$.

By lemma (2.2) we get, $Av = v$. (3.11)

Since B is continuous, then we have $B^2w_{n+1} = Bv$. (3.12)

As (B, T) is compatible of type (α) we get, $TBw_{n+1} \rightarrow Bv$. (3.13)

Step-(3): Taking $x = u_n$ and $y = Bw_{n+1}$ in (3.5) we get,

$[1 + aM(Su_n, TBw_{n+1}, kt)]*M(Au_n, BBw_{n+1}, kt) \geq a\text{Min}\{M(BBw_{n+1}, TBw_{n+1}, kt),$

$M(Au_n, Su_n, kt), M(Au_n, TBw_{n+1}, 2kt), M(BBw_{n+1}, Su_n, 2kt)\}+\text{Min}\{M(Au_n, Su_n, t),$

$M(Su_n, TBw_{n+1}, t), M(BBw_{n+1}, TBw_{n+1}, t), M(Au_n, TBw_{n+1}, 2t), M(BBw_{n+1}, Su_n, 2t)\}$

$[1 + aM(v, Bv, kt)]*M(v, Bv, kt) \geq a\text{Min}\{M(Bv, Bv, kt), M(v, v, kt), M(v, Bv, 2kt),$

$M(Bv, v, 2kt)\}+\text{Min}\{M(v, v, t), M(v, Bv, t), M(Bv, Bv, t), M(v, Bv, 2t), M(Bv, v, 2t)\}$

$M(v, Bv, kt) + aM(v, Bv, kt)*M(v, Bv, kt) \geq a\text{Min}\{1, 1, M(v, Bv, 2kt), M(Bv, v, 2kt)\}$

$+\text{Min}\{1, M(v, Bv, t), 1, M(v, Bv, 2t), M(Bv, v, 2t)\}$

$M(v, Bv, kt) + aM(v, Bv, kt) \geq aM(v, Bv, kt) + M(v, Bv, t)$

and $[1 + aN(Su_n, TBw_{n+1}, kt)]\Diamond N(Au_n, BBw_{n+1}, kt) \leq a\text{Max}\{N(BBw_{n+1}, TBw_{n+1}, kt),$

$N(Au_n, Su_n, kt), N(Au_n, TBw_{n+1}, 2kt), N(BBw_{n+1}, Su_n, 2kt)\}+\text{Max}\{N(Au_n, Su_n, t),$

$N(Su_n, TBw_{n+1}, t), N(BBw_{n+1}, TBw_{n+1}, t), N(Au_n, TBw_{n+1}, 2t), N(BBw_{n+1}, Su_n, 2t)\}$

$[1 + aN(v, Bv, kt)]\Diamond N(v, Bv, kt) \leq a\text{Max}\{N(Bv, Bv, kt), N(v, v, kt), N(v, Bv, 2kt),$

$N(Bv, v, 2kt)\}+\text{Max}\{N(v, v, t), N(v, Bv, t), N(Bv, Bv, t), N(v, Bv, 2t), N(Bv, v, 2t)\}$

$N(v, Bv, kt) + aN(v, Bv, kt)\Diamond N(v, Bv, kt) \leq a\text{Max}\{0, 0, N(v, Bv, 2kt), N(Bv, v, 2kt)\}$

$+\text{Max}\{0, N(v, Bv, t), 0, N(v, Bv, 2t), N(Bv, v, 2t)\}$

$N(v, Bv, kt) + aN(v, Bv, kt) \leq aN(v, Bv, kt) + N(v, Bv, t)$

That is, $M(v, Bv, kt) \geq M(v, Bv, t)$ and $N(v, Bv, kt) \leq N(v, Bv, t)$.

By lemma (2.2) we get, $Bv = v$. Thus $Av = Bv = v$. (3.14)

Step-(4): Taking $x = u_n, y = Tw_{n+1}$ in (3.5) we get,

$[1 + aM(Su_n, TTw_{n+1}, kt)]*M(Au_n, BTw_{n+1}, kt) \geq a\text{Min}\{M(BTw_{n+1}, TTw_{n+1}, kt),$

$M(Au_n, Su_n, t), M(Au_n, TTw_{n+1}, 2kt), M(BTw_{n+1}, Su_n, 2kt)\}+\text{Min}\{M(Au_n, Su_n, t),$

$M(Su_n, TTw_{n+1}, t), M(BTw_{n+1}, TTw_{n+1}, t), M(Au_n, TTw_{n+1}, 2t), M(BTw_{n+1}, Su_n, 2t)\}$

$[1 + aM(v, Tv, kt)]*M(v, Tv, kt) \geq a\text{Min}\{M(Tv, Tv, kt), M(v, v, t), M(v, Tv, 2kt),$

$M(Tv, v, 2kt)\}+\text{Min}\{M(v, v, t), M(v, Tv, t), M(Tv, Tv, t), M(v, Tv, 2t), M(Tv, v, 2t)\}$

$M(v, Tv, kt) + aM(v, Tv, kt)*M(v, Tv, kt) \geq a\text{Min}\{1, 1, M(v, Tv, 2kt), M(Tv, v, 2kt)\}$

$+\text{Min}\{1, M(v, Tv, t), 1, M(v, Tv, 2t), M(Tv, v, 2t)\}$

$M(v, Tv, kt) + aM(v, Tv, kt) \geq aM(v, Tv, kt) + M(v, Tv, t)$

and $[1 + aN(Su_n, TTw_{n+1}, kt)]\Diamond N(Au_n, BTw_{n+1}, kt) \leq a\text{Max}\{N(BTw_{n+1}, TTw_{n+1}, kt),$

$N(Au_n, Su_n, t), N(Au_n, TTw_{n+1}, 2kt), N(BTw_{n+1}, Su_n, 2kt)\}+\text{Max}\{N(Au_n, Su_n, t),$

$N(Su_n, TTw_{n+1}, t), N(BTw_{n+1}, TTw_{n+1}, t), N(Au_n, TTw_{n+1}, 2t), N(BTw_{n+1}, Su_n, 2t)\}$

$[1 + aN(v, Tv, kt)]\Diamond N(v, Tv, kt) \leq a\text{Max}\{N(Tv, Tv, kt), N(v, v, t), N(v, Tv, 2kt),$

$N(Tv, v, 2kt)\}+\text{Max}\{N(v, v, t), N(v, Tv, t), N(Tv, Tv, t), N(v, Tv, 2t), N(Tv, v, 2t)\}$

$N(v, Tv, kt) + aN(v, Tv, kt)\Diamond N(v, Tv, kt) \leq \text{aMax}\{0, 0, N(v, Tv, 2kt), N(Tv, v, 2kt)\}$

$+\text{Max}\{0, N(v, Tv, t), 0, N(v, Tv, 2t), N(Tv, v, 2t)\}$

$N(v, Tv, kt) + aN(v, Tv, kt) \leq aN(v, Tv, kt) + N(v, Tv, t)$

That is, $M(v, Tv, kt) \geq M(v, Tv, t)$ and $N(v, Tv, kt) \leq N(v, Tv, t)$.

By lemma (2.2) we get, $Tv = v$. Thus $Av = Bv = Tv = v$.

Step-(5): As $AT(X) \subseteq BS(X)$, there exists $z \in X$ such that $v = ATv = STz$.

Writing $Tz = u$. Therefore $v = Su$. (3.15)

Now, taking $x = u$ and $y = v$ in (3.5) we get,

$[1 + aM(Su, Tv, kt)]*M(Au, Bv, kt) \geq a\text{Min}\{M(Bv, Tv, kt), M(Au, Su, kt), M(Au, Tv, 2kt),$

$M(Bv, Su, 2kt)\}+\text{Min}\{M(Au, Su, t), M(Su, Tv, t), M(Bv, Tv, t), M(Au, Tv, 2t), M(Bv, Su, 2t)\}$

$[1 + aM(v, v, kt)]*M(Au, v, kt) \geq a\text{Min}\{M(v, v, kt), M(Au, v, kt), M(Au, v, 2kt),$

$M(v, v, 2kt)\}+\text{Min}\{M(Au, v, t), M(v, v, t), M(v, v, t), M(Au, v, 2t), M(v, v, 2t)\}$

$M(Au, v, kt) + aM(Au, v, kt)*M(Au, v, kt) \geq a\text{Min}\{1, M(Au, v, kt), M(Au, v, 2kt), 1\}$

$+\text{Min}\{M(Au, v, t), 1, 1, M(Au, v, 2t), M(v, v, 2t)\}$

$M(Au, v, kt) + aM(Au, v, kt) \geq aM(Au, v, kt) + M(Au, v, t)$

and $[1 + aN(Su, Tv, kt)]\Diamond N(Au, Bv, kt) \leq a\text{Max}\{N(Bv, Tv, kt), N(Au, Su, kt), N(Au, Tv, 2kt),$

$N(Bv, Su, 2kt)\}+\text{Max}\{N(Au, Su, t), N(Su, Tv, t), N(Bv, Tv, t), N(Au, Tv, 2t), N(Bv, Su, 2t)\}$

$[1 + aN(v, v, kt)]\Diamond N(Au, v, kt) \leq a\text{Max}\{N(v, v, kt), N(Au, v, kt), N(Au, v, 2kt),$

$N(v, v, 2kt)\}+\text{Max}\{N(Au, v, t), N(v, v, t), N(v, v, t), N(Au, v, 2t), N(v, v, 2t)\}$

$N(Au, v, kt) + aN(Au, v, kt)\Diamond N(Au, v, kt) \leq a\text{Max}\{0, N(Au, v, kt), N(Au, v, 2kt), 0\}$

$+\text{Max}\{N(Au, v, t), 0, 0, N(Au, v, 2t), N(v, v, 2t)\}$

$N(Au, v, kt) + aN(Au, v, kt) \leq aN(Au, v, kt) + N(Au, v, t)$

That is, $M(Au, v, kt) \geq M(Au, v, t)$ and $N(Au, v, kt) \leq N(Au, v, t)$.

By lemma (2.1) we get, $Au = v$. Now $Au = Su = v$.

As (A, S) is compatible of type (α) and so is weak compatible.

Hence $v = Av = Sv$. Therefore $Av = Bv = Sv = Tv = v$.

Case-(II): S and T are continuous. The continuity of S implies $S^2u_n \to Sv$. (3.16)

As (A, S) is compatible of type (α), by (3.6) we get, $ASu_n \to Sv$. (3.17)

Step-(6): Taking $x = Su_n$ and $y = w_{n+1}$ in (3.5) we get,

$[1 + aM(SSu_n, Tw_{n+1}, kt)]*M(ASu_n, Bw_{n+1}, kt) \geq a\text{Min}\{M(Bw_{n+1}, Tw_{n+1}, kt), M(ASu_n, SSu_n, kt),$

$M(ASu_n, Tw_{n+1}, 2kt), M(Bw_{n+1}, SSu_n, 2kt)\}+\text{Min}\{M(ASu_n, SSu_n, t), M(SSu_n, Tw_{n+1}, t),$

$M(Bw_{n+1}, Tw_{n+1}, t), M(ASu_n, Tw_{n+1}, 2t), M(Bw_{n+1}, SSu_n, 2t)\}$

$[1 + aM(Sv, Sv, kt)]*M(Sv, v, kt) \geq a\text{Min}\{M(v, v, kt), M(Sv, Sv, t), M(Sv, v, 2kt),$

$M(v, Sv, 2kt)\}+\text{Min}\{M(Sv, Sv, t), M(Sv, v, t), M(v, v, t), M(Sv, v, 2t), M(v, Sv, 2t)\}$

$M(Sv, v, kt) + aM(Sv, v, kt)*M(Sv, v, kt) \geq a\text{Min}\{1, 1, M(Sv, v, 2kt), M(v, Sv, 2kt)\}$

$+\text{Min}\{1, M(Sv, v, t), 1, M(Sv, v, 2t), M(v, Sv, 2t)\}$

$M(Sv, v, kt) + aM(Sv, v, kt) \geq aM(Sv, v, kt) + M(v, Sv, t)$

and $[1 + aN(SSu_n, Tw_{n+1}, kt)]\Diamond N(ASu_n, Bw_{n+1}, kt) \leq a\text{Max}\{N(Bw_{n+1}, Tw_{n+1}, kt),$

$N(ASu_n, SSu_n, kt), N(ASu_n, Tw_{n+1}, 2kt), N(Bw_{n+1}, SSu_n, 2kt)\}+\text{Max}\{N(ASu_n, SSu_n, t),$

$N(SSu_n, Tw_{n+1}, t), N(Bw_{n+1}, Tw_{n+1}, t), N(ASu_n, Tw_{n+1}, 2t), N(Bw_{n+1}, SSu_n, 2t)\}$

$[1 + aN(Sv, Sv, kt)]\Diamond N(Sv, v, kt) \leq a\text{Max}\{N(v, v, kt), N(Sv, Sv, t), N(Sv, v, 2kt),$

$N(v, Sv, 2kt)\}+\text{Max}\{N(Sv, Sv, t), N(Sv, v, t), N(v, v, t), N(Sv, v, 2t), N(v, Sv, 2t)\}$

$N(Sv, v, kt) + aN(Sv, v, kt)\Diamond N(Sv, v, kt) \leq a\text{Max}\{0, 0, N(Sv, v, 2kt), N(v, Sv, 2kt)\}$

$+\text{Max}\{0, N(Sv, v, t), 0, N(Sv, v, 2t), N(v, Sv, 2t)\}$

$N(Sv, v, kt) + aN(Sv, v, kt) \leq aN(Sv, v, kt) + N(v, Sv, t)$

That is, $M(Sv, v, kt) \geq M(v, Sv, t)$ and $N(Sv, v, kt) \leq N(v, Sv, t)$.

By lemma (2.2) we get, $Sv = v$. (3.18)

Step-(7): Taking $x = v$ and $y = w_{n+1}$ in (3.5) we get,

$[1 + aM(Sv, Tw_{n+1}, kt)]*M(Av, Bw_{n+1}, kt) \geq a\text{Min}\{M(Bw_{n+1}, Tw_{n+1}, kt), M(Av, Sv, kt),$

$M(Av, Tw_{n+1}, 2kt), M(Bw_{n+1}, Sv, 2kt)\}+\text{Min}\{M(Av, Sv, t), M(Sv, Tw_{n+1}, t),$

$M(Bw_{n+1}, Tw_{n+1}, t),M(Av, Tw_{n+1}, 2t), M(Bw_{n+1}, Sv, 2t)\}$

$[1 + aM(v, v, kt)]*M(Av, v, kt) \geq a\text{Min}\{M(v, v, t), M(Av, v, kt), M(Av, v, 2kt), M(v, v, 2kt)\}$

$+\text{Min}\{M(Av, v, t), M(v, v, t), M(v, v, t), M(Av, v, 2t), M(v, v, 2t)\}$

$M(Av, v, kt) + aM(v, v, kt)*M(Av, v, kt) \geq a\text{Min}\{1, M(Av, v, 2kt), M(Av, v, 2kt), 1\}$

$+\text{Min}\{M(Av, v, t), 1, 1, M(Av, v, 2t), 1\}$

$M(Av, v, kt) + aM(Av, v, kt) \geq aM(Av, v, kt) + M(Av, v, t)$

and $[1 + aN(Sv, Tw_{n+1}, kt)]\Diamond N(Av, Bw_{n+1}, kt) \leq a\text{Max}\{N(Bw_{n+1}, Tw_{n+1}, kt), N(Av, Sv, kt),$

$N(Av, Tw_{n+1}, 2kt), N(Bw_{n+1}, Sv, 2kt)\}+\text{Max}\{N(Av, Sv, t), N(Sv, Tw_{n+1}, t),$

$N(Bw_{n+1}, Tw_{n+1}, t), N(Av, Tw_{n+1}, 2t), N(Bw_{n+1}, Sv, 2t)\}$

$[1 + aN(v, v, kt)]\Diamond N(Av, v, kt) \le a\text{Max}\{N(v, v, t), N(Av, v, kt), N(Av, v, 2kt),$

$N(v, v, 2kt)\}+\text{Max}\{N(Av, v, t), N(v, v, t), N(v, v, t), N(Av, v, 2t), N(v, v, 2t)\}$

$N(Av, v, kt) + aN(v, v, kt)\Diamond N(Av, v, kt) \le a\text{Max}\{0, N(Av, v, 2kt), N(Av, v, 2kt), 0\}$

$+\text{Max}\{N(Av, v, t), 0, 0, N(Av, v, 2t), 0\}$

$N(Av, v, kt) + aN(Av, v, kt) \le aN(Av, v, kt) + N(Av, v, t)$

That is, $M(Av, v, kt) \ge M(Av, v, t)$ and $N(Av, v, kt) \le N(Av, v, t)$.

By lemma (2.2) we get, $Av = v$. Thus, $Sv = Av = v$. (3.19)

The continuity of T gives, $T^2w_{n+1} \to Tv$. (3.20)

As (B, T) is compatible of type (α), then by (3.6) we get $BTw_{n+1} \to Tv$. (3.21)

Step-(8): Taking $x = u_n$ and $y = Tw_{n+1}$ in (3.5) we get,

$[1 + aM(Su_n, TTw_{n+1}, kt)]*M(Au_n, BTw_{n+1}, kt) \ge a\text{Min}\{M(BTw_{n+1}, TTw_{n+1}, kt), M(Au_n, Su_n, kt),$

$M(Au_n, TTw_{n+1}, 2kt), M(BTw_{n+1}, Su_n, 2kt)\}+\text{Min}\{M(Au_n, Su_n, t), M(Su_n, TTw_{n+1}, t),$

$M(BTw_{n+1}, TTw_{n+1}, t), M(Au_n, Tw_{n+1}, 2t), M(BTw_{n+1}, Su_n, 2t)\}$

$[1 + aM(v, Tv, kt)]*M(v, Tv, kt) \ge a\text{Min}\{M(Tv, Tv, kt), M(v, v, kt), M(v, Tv, 2kt),$

$M(Tv, v, 2kt)\}+\text{Min}\{M(v, v, t), M(v, Tv, t), M(Tv, Tv, t), M(v, Tv, 2t), M(Tv, v, 2t)\}$

$M(v, Tv, kt) + aM(v, Tv, kt)*M(v, Tv, kt) \ge a\text{Min}\{1, 1, M(v, Tv, 2kt), M(Tv, v, 2kt)\}$

$+\text{Min}\{1, M(v, Tv, t), 1, M(v, Tv, 2t), M(Tv, v, 2t)\}$

$M(v, Tv, kt) + aM(v, Tv, kt) \ge aM(v, Tv, kt) + M(v, Tv, t)$

and $[1 + aN(Su_n, TTw_{n+1}, kt)]\Diamond N(Au_n, BTw_{n+1}, kt) \le a\text{Max}\{N(BTw_{n+1}, TTw_{n+1}, kt),$

$N(Au_n, Su_n, kt), N(Au_n, TTw_{n+1}, 2kt), N(BTw_{n+1}, Su_n, 2kt)\}+\text{Max}\{N(Au_n, Su_n, t),$

$N(Su_n, TTw_{n+1}, t), N(BTw_{n+1}, TTw_{n+1}, t), N(Au_n, Tw_{n+1}, 2t), N(BTw_{n+1}, Su_n, 2t)\}$

$[1 + aN(v, Tv, kt)]\Diamond N(v, Tv, kt) \le a\text{Max}\{N(Tv, Tv, kt), N(v, v, kt), N(v, Tv, 2kt),$

$N(Tv, v, 2kt)\}+\text{Max}\{N(v, v, t), N(v, Tv, t), N(Tv, Tv, t), N(v, Tv, 2t), N(Tv, v, 2t)\}$

$N(v, Tv, kt) + aN(v, Tv, kt)\Diamond N(v, Tv, kt) \le a\text{Max}\{0, 0, N(v, Tv, 2kt), N(Tv, v, 2kt)\}$

$+\text{Max}\{0, N(v, Tv, t), 0, N(v, Tv, 2t), N(Tv, v, 2t)\}$

$N(v, Tv, kt) + aN(v, Tv, kt) \leq aN(v, Tv, kt) + N(v, Tv, t)$

That is, $M(v, Tv, kt) \geq M(v, Tv, t)$ and $N(v, Tv, kt) \leq N(v, Tv, t)$.

By lemma (2.2) we get, $Tv = v$. (3.22)

Step-(9): Taking $x = v$ and $y = v$ in (3.5) we get,

$[1 + aM(Sv, Tv, kt)]*M(Av, Bv, kt) \geq a\text{Min}\{M(Bv, Tv, kt), M(Av, Sv, kt), M(Av, Tv, 2kt),$

$M(Bv, Sv, 2kt)\}+\text{Min}\{M(Av, Sv, t), M(Sv, Tv, t), M(Bv, Tv, t), M(Av, Tv, 2t), M(Bv, Sv, 2t)\}$

$[1 + aM(v, v, kt)]*M(v, Bv, kt) \geq a\text{Min}\{M(Bv, v, kt), M(v, v, kt), M(v, v, 2kt),$

$M(Bv, v, 2kt)\}+\text{Min}\{M(v, v, t), M(v, v, t), M(Bv, v, t), M(v, v, 2t), M(Bv, v, 2t)\}$

$M(v, Bv, kt) + aM(v, Bv, kt) \geq a\text{Min}\{M(Bv, v, kt), 1, 1, M(Bv, v, 2kt)\}$

$+\text{Min}\{1, 1, M(Bv, v, t), 1, M(Bv, v, 2t)\}$

$M(v, Bv, kt) + aM(v, Bv, kt) \geq aM(Bv, v, kt) + M(Bv, v, t)$

and $[1 + aN(Sv, Tv, kt)]\Diamond N(Av, Bv, kt) \leq a\text{Max}\{N(Bv, Tv, kt), N(Av, Sv, kt), N(Av, Tv, 2kt),$

$N(Bv, Sv, 2kt)\}+\text{Max}\{N(Av, Sv, t), N(Sv, Tv, t), N(Bv, Tv, t), N(Av, Tv, 2t), N(Bv, Sv, 2t)\}$

$[1 + aN(v, v, kt)]\Diamond N(v, Bv, kt) \leq a\text{Max}\{N(Bv, v, kt), N(v, v, kt), N(v, v, 2kt),$

$N(Bv, v, 2kt)\}+\text{Max}\{N(v, v, t), N(v, v, t), N(Bv, v, t), N(v, v, 2t), N(Bv, v, 2t)\}$

$N(v, Bv, kt) + aN(v, Bv, kt) \leq a\text{Max}\{N(Bv, v, kt), 0, 0, N(Bv, v, 2kt)\}$

$+\text{Max}\{0, 0, N(Bv, v, t), 0, N(Bv, v, 2t)\}$

$N(v, Bv, kt) + aN(v, Bv, kt) \leq aN(Bv, v, kt) + N(Bv, v, t)$

That is, $M(v, Bv, kt) \geq M(Bv, v, t)$ and $N(v, Bv, kt) \leq N(Bv, v, t)$.

By lemma (2.2) we get, $Bv = v$. Therefore $Bv = Tv = v$.

Thus in this case also $Av = Bv = Sv = Tv = v$.

Hence in both the cases, we get v is a common fixed point in X.

Uniqueness: Let v_1 be another common fixed point of A, B, S and T. Then $Av_1 = Bv_1 = Sv_1 = Tv_1 = v_1$. Now we assume that, $v \neq v_1$. Taking $x = v$ and $y = v_1$ in (3.5), we get

$[1 + aM(Sv, Tv_1, kt)]*M(Av, Bv_1, kt) \geq a\text{Min}\{M(Bv_1, Tv_1, kt), M(Av, Sv, kt), M(Av, Tv_1, 2kt),$

$M(Bv_1, Sv, 2kt)\}+\text{Min}\{M(Av, Sv, t), M(Sv, Tv_1, t), M(Bv_1, Tv_1, t), M(Av, Tv_1, 2t),$

$M(Bv_1, Sv, 2t)\}$

$[1 + aM(v, v_1, kt)]*M(v, v_1, kt) \geq a\text{Min}\{M(v_1, v_1, kt), M(v, v, kt), M(v, v_1, 2kt),$

$M(v_1, v, 2kt)\}$+Min$\{M(v, v, t), M(v, v_1, t), M(v_1, v_1, t), M(v, v_1, 2t), M(v_1, v, 2t)\}$

$M(v, v_1, kt) + aM(v, v_1, kt) * M(v, v_1, kt) \geq aMin\{1, 1, M(v, v_1, 2kt), M(v_1, v, 2kt)\}$

+Min$\{1, M(v, v_1, t), 1, M(v, v_1, 2t), M(v_1, v, 2t)\}$

$M(v, v_1, kt) + aM(v, v_1, kt) \geq aM(v, v_1, kt) + M(v, v_1, t)$

and $[1 + aN(Sv, Tv_1, kt) \Diamond N(Av, Bv_1, kt) \leq aMax\{N(Bv_1, Tv_1, kt), N(Av, Sv, kt)$,

$N(Av, Tv_1, 2kt), N(Bv_1, Sv, 2kt)\}$+Max$\{N(Av, Sv, t), N(Sv, Tv_1, t), N(Bv_1, Tv_1, t)$,

$N(Av, Tv_1, 2t), N(Bv_1, Sv, 2t)\}$

$[1 + aN(v, v_1, kt)] \Diamond N(v, v_1, kt) \leq aMax\{N(v_1, v_1, kt), N(v, v, kt), N(v, v_1, 2kt)$,

$N(v_1, v, 2kt)\}$+Max$\{N(v, v, t), N(v, v_1, t), N(v_1, v_1, t), N(v, v_1, 2t), N(v_1, v, 2t)\}$

$N(v, v_1, kt) + aN(v, v_1, kt) \Diamond N(v, v_1, kt) \leq aMax\{0, 0, N(v, v_1, 2kt), N(v_1, v, 2kt)\}$

+Max$\{0, N(v, v_1, t), 0, N(v, v_1, 2t), N(v_1, v, 2t)\}$

$N(v, v_1, kt) + aN(v, v_1, kt) \leq aN(v, v_1, kt) + N(v, v_1, t)$

That is, $M(v, v_1, kt) \geq M(v, v_1, t)$ and $N(v, v_1, kt) \leq N(v, v_1, t)$.

By lemma (2.2) we get, $v = v_1$. This is contradiction.

Hence v is the unique common fixed point of A, B, S and T.

Example (3.1): Let $X = \{\frac{1}{n}\, n \in N\} \cup \{0\}$ with the metric d defined by $d(x, y) = |x-y|$ and for each $t>0$, let M_d, N_d be fuzzy sets on $X^2 \times [0, \infty)$, which are defined as follows

$M_d(x, y, t) = \frac{t}{t+d(x,y)}$ and $N_d(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$

For all $x, y \in X$. Clearly, $(X, M_d, N_d, *, \Diamond)$ is a complete intuitionistic fuzzy metric space where $*$, $\Diamond$ are defined by $a*b = \min\{a, b\}$ for all $a, b \in [0, 1]$. Let A, B, S and T be maps from X into itself, which are defined by

$Ax = \frac{x}{6}$, $Bx = 0$, $Sx = \frac{x}{3}$, $Tx = x$ for all $x \in X$. Then

$AT(X) \cup BS(X) = \{\frac{1}{6n} | n \in N\} \cup \{0\} \subset \{\frac{1}{3n} | n \in N\} \cup \{0\} = ST(X)$.

Furthermore, $ST = TS$ and S, T are continuous. If we take $k = \frac{1}{2}$ and $t = 1$, the condition (3.5) of theorem (3.1) is satisfied.

Moreover, A, S are type (α) compatible if $\lim_{n\to\infty} x_n = 0$ where $\{x_n\} \subset X$ such that,

$lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 0$ for some $0 \in X$.

Similarly, B, T are type (α) compatible. Thus,

$M(0, B0, kt) + aM(0, B0, kt) \geq aM(0, B0, kt) + M(0, B0, t)$

and $N(0, B0, kt) + aM(0, B0, kt) \leq aN(0, B0, kt) + N(0, B0, t)$.

Therefore, $M(0, B0, kt) \geq M(0, B0, t)$ and $N(0, B0, kt) \leq N(0, B0, t)$ for all $t>0$ and $k \in (0, 1)$. Thus, $0 = B0$. Similarly, we obtain $0 = A0$.

Therefore, 0 is a common fixed point of A, B, S and T.

Let w, be another fixed point of A, B, S and T. Then

$$[1 + aM(0, w, kt)]*M(0, w, kt) \geq a\text{Min}\{M(w, w, kt), M(0, 0, kt), M(0, w, 2kt), M(w, 0, 2kt)\} + \text{Min}\{M(0, 0, t), M(0, w, t), M(w, w, t), M(0, w, 2t), M(w, 0, 2t)\}$$

$M(0, w, kt) + aM(0, w, kt) \geq aM(0, w, kt) + M(0, w, t)$

and $[1 + aN(0, w, kt)]\Diamond N(0, w, kt) \leq a\text{Max}\{N(w, w, kt), N(0, 0, kt), N(0, w, 2kt), N(w, 0, 2kt)\} + \text{Max}\{N(0, 0, t), N(0, w, t), N(w, w, t), N(0, w, 2t), N(w, 0, 2t)\}$
$N(0, w, kt) + aN(0, w, kt) \leq aN(0, w, kt) + N(0, w, t)$.

Therefore, $M(0, w, kt) \geq M(0, w, t)$ and $N(0, w, kt) \leq N(0, w, t)$ for all $t>0$ and $k \in (0, 1)$.

Therefore, $0 = w$. Thus, A, B, S and T have a unique common fixed point 0.

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