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Research Paper

A Fuzzy Approach for Solving Bi-Level Programming Problem of Banking Crisis Management

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Abstract: *Banking risk management as a decision making challenge is basically a complex structure with non crisp input factors. Based on the strength of those attributes it is a reasonable way for fast and human like decision to group the factors, and to use the fuzzy approach in banking risk management modeling. Bi-level programming problem of banking crisis management with linear or non-linear constraints, and in which the objective function at each level are to be maximized. The bi-level programming problem can be thought as a static version of the Stackelberg strategy, which is used leader-follower game in which a Stackelberg strategy is used by the leader, or the higher-level decision-maker (manager), given the rational reaction of the follower, or the lower-level decision-maker (follower). This paper proposes a two-planner model and a solution method for solving this problem. This method uses the concept of tolerance membership function to develop a fuzzy Max-Min decision model for generating Pareto optimal solution for this problem; an illustrative numerical example is given to demonstrate the obtained results.*

Keywords: Risk management, bi –level, Fuzzy programming, Pareto optimal solution, Stackelberg game.

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1. Introduction

Calculating total cast of bank resources procurement methods which include current -free loan deposit, saving interest-free loan deposit, regular and net short-term investment deposit, long-term investment deposit and surety bond cash deposit and presenting their optimal integration require precise scientific studies.

The economical crisis situations and the complex environmental and societal processes over the past years indicate the need for new mathematical model constructions to predict their effects. The health diagnostic as a multi-parameter and multi-criteria decision making system is a risk model should be managed.

Bi-level programming is a powerful and robust technique for solving hierarchical decision making problem. It has been applied in many real life problems such as agriculture, bio-fuel production, economic systems, finance, engineering, banking, management sciences, transportation problem ([2], [7], [10], [13]).

We can also site the Stackelberg duopoly: two firms supply homogeneous goods to a market, and consequently the predominant firm first its level of supply, and then the over firm determines that of itself after it realizes that of the predominant firm.

Risk management modeling as a complex, multi-parametrical problem is one of the main research fields in the world today, from the micro-communities, families to the macro society structures and global phenomena of nature monitoring.

Statistical methods-based reasoning models in crisis situations need long-time experiments and enough reliable data elaborated by experts. Additionally, they are time and computing-demanding. The problems to be solved are full of uncertainties, and complexity of the systems increases the runtime factor of the decision process. Considering all those conditions fuzzy set theory helps manage complexity and uncertainties, and represents the inputs and outputs of the model in an emphatic form.

Recently, notable studies have been done in the area of bi-level programming and banking crisis management optimization problems ([8], [12]).

In [8] Etoa presented a smoothing sequential quadratic programming to determine a solution of a convex quadratic bi-level programming problem. Pramanik and Dey presented in [11] a fuzzy goal programming approach for bi-level linear fractional programming problem with a single decision maker at the upper level and a single decision maker at the lower level.

Each level has single objective function, which are fractional in nature and the system constraints are linear functions. The approach first construct fractional membership functions by determining individual best solution of the objective functions subject to the system constraints. The fractional membership functions are then transformed into equivalent linear membership functions by first order Taylor polynomial series.

In [12], Pramanik and Dey presented fuzzy goal programming approach to quadratic bi-level programming problem, which construct the quadratic membership functions by determining individual best solutions of the quadratic objective functions subject to the system constraints.

The quadratic membership functions are then transformed into equivalent linear membership functions by first order Taylor series approximation at the individual best solution point. Then fuzzy goal programming approach is used for achieving highest degree of each of the membership goals by minimizing deviational variables.

Arora and Gupta [2] presented interactive fuzzy goal programming approach for linear bi-level programming problem with the characteristics of dynamic programming. Satisfactory solution is derived by updating the satisfactory degree of the decision makers with the consideration of overall satisfactory balance between both levels.

2. Bi-Level Banking Risk Management Systems

The steps of the banking risk management model constructions are following the systematic approach, the steps of the problem solving are as listed: the banking risk factor identification, the qualitative or quantitative description of their effects on the environment, the development of response actions to these risks, and if possible, and trying to increase the effects of them.

Based on those ideas a banking risk management system can be built up as a bi-level system of the risk factors (inputs), banking risk management actions (decision making system) and direction or directions for the next level of risk situation solving algorithm.

Actually, those directions are risk factors for the action on the next level of banking risk management process. To realize this risk factors in a complex system are grouped to the risk relevant events or decision step.

Risk of banking management is the identification, assessment, and prioritization of risks, defined as the effects of uncertainty of objectives, whether positive or negative, followed by the coordinated and economical application of bank resources to minimize, monitor, and control the probability and/or impact of unfortunate events [6].

The techniques used in banking risk management have been taken from other areas of system management. Information technology, the availability of resources, and other facts have helped to develop the new risk management with the methods to identify measure and manage the risks, thereby reducing the potential for unexpected loss or harm.

Generally, banking risk management process involves the following main stages. The first step is the identification of risks and potential risks to the bank system operation at all levels. Evaluation, the measure and structural systematization of the identified risks, is the next step. Measurement is defined by how serious the risks are in terms of consequences and the likelihood of occurrence.

It can be a qualitative or quantitative description of their effects on the bank environment. Plan and control are the next stages to prepare the risk management system.

This can include the development of response actions to these risks, and the applied decision or reasoning method. Monitoring and review, as the next stage, is important if we are to have bank system with feedback, and the risk management system is open to improvement.

This will ensure that the banking risk management process is dynamic and continuous, with correct verification and validity control. The review process includes the possibility of new additional risks and new forms of risk description. In the future the role of complex risk management will be to try to increase the damaging effects of risk factors.

3. Problem Formulation and Solution Concept

Let $x_i \in R^{n_i}, (i = 1,2)$ be a vector variables indicating the first decision level's bank choice and the second decision level's bank choice, $n_i \geq 1, (i = 1,2)$.

Let $F_i : R^{n_i} \to R^{N_i}, (i = 1,2)$ be the first level objective function (manager), and the second level objective function (follower), respectively. Let the manager and follower have N_1 and N_2 objective function, respectively.

Therefore, the bi-level risk management problem may be formulated as follows:

[Manger Level]

$$\underset{x_1}{Max}\, F_1(x_1, x_2), \tag{1}$$

where x_2 solves

[Follower Level]

$$Max\, F_2(x_1, x_2), \tag{2}$$

Subject to

$$G = \{(x_1, x_2) \mid g_i(x_1, x_2) \le 0, i = 1,2,\dots,m, \\ x_1, x_2 \ge 0, \} . \tag{3}$$

Where G is the bi-level non-convex constraint set. F_1 and F_2 are non-linear functions.

Definition 1: *For any* $x_1(x_1 \in G_1 = \{x_1 | (x_1, x_2) \in G\})$ *given by manager, if the decision-making variable* $x_2(x_2 \in G_2 = \{x_2 | (x_1, x_2) \in G\})$ *is the Pareto optimal solution of the follower, then* (x_1, x_2) *is a feasible solution of problem.*

Definition 2: *If* (x_1^*, x_2^*) *is a feasible solution of the problem; no other feasible solution* $(x_1, x_2) \in G$ *exists, such that* $F_1(x_1^*, x_2^*) \le F_1(x_1, x_2)$ *; so* (x_1^*, x_2^*) *is the Pareto optimal solution of the problem.*

4. Fuzzy Decision Models for BLI –NLP Problem

To solve the banking risk management problem by adopting, the leader-follower Stakelberg and the well-known fuzzy decision model of Sakawa ([10], [11]). One first gets the satisfactory integer solution that is acceptable to manager, and then gives the manager decision variable and goal with some leeway to the follower for him/her to seek the optimal solution, and to arrive at the solution which is closest to the optimal solution of the manager.

4.1 Manager Problem

First, the manager solves the following Problem:

$$\underset{x}{Max}\, F_1(x) \tag{4}$$

Subject to

$$x \in G \quad .$$

To build membership function, goals and tolerances should be determined first. However, they could hardly be determined without meaningful supporting data.

We should first find the individual best solution $\left(F_1^*\right)$ and individual worst solution $\left(F_1^-\right)$ of (4), where

$$F_1^* = \underset{x \in G}{Max}\, F_1(x) \quad , \tag{5}$$

Goals and tolerances can then be reasonably set for individual solution and the difference of the best and worst solution, respectively. This data can then be formulated as the following membership function of fuzzy set theory [10]:

$$\mu\ [F_1(x)] = \begin{cases} 1 & if \quad F_1(x) > F_1^* \\ \dfrac{F_1(x) - F_1^-}{F_1^* - F_1^-} & if \quad F_1^- \le F_1(x) \le F_1^* \\ 0 & if \quad F_1^- \ge F_1(x). \end{cases} \tag{6}$$

Now, we can get the solution of the MANAGER problem by solving the following mixed integer Tchebycheff problem:

$$Max\, \lambda \tag{7}$$

Subject to
$x \in G,$
$\mu\ [F_1(x)] \ge \lambda,$
$\lambda \in [0,1].$

By the branch and bound technique, the manager solution is assumed to be

$$\left[x_1^H, x_2^H,\ F_1^H,\ \lambda^H\ (Satisfactory\, Level)\right]?$$

4.2 Follower Problem

Second, in the same way, the follower independently solves:

$$\underset{x}{Max}\, F_2(x), \tag{8}$$

Subject to
$x \in G$.

The individual best solution $\left(F_2^*\right)$ and individual worst solution $\left(F_1^-\right)$ of (8) are:

$$F_2^* = \underset{x \in G}{Max}\, F_2(x) \quad , \ F_2^-(x) = \underset{x \in G}{Min}\, F_2(x) \tag{9}$$

This information can then be formulated as the following membership function of Fuzzy set theory:

$$\mu\ [F_2(x)] = \begin{cases} 1 & if \quad F_2(x) > F_2^* \\ \dfrac{F_2(x) - F_2^-}{F_2^* - F_2^-} & if \quad F_2^- \le F_2(x) \le F_2^* \\ 0 & if \quad F_2^- \ge F_2(x). \end{cases} \tag{10}$$

Now the solution of the LLDM can be obtained by solving the following mixed integer Tchebycheff problem:

$$Max\,\beta\,, \tag{11}$$

Subject to
$x \in G,$
$\mu\ [F_2(x)] \ge \beta,$
$\beta \in [0,1].$

By the branch and bound technique, the LLDM solution is assumed to be

$$\left[x_1^L, x_2^L,\ F_2^L,\ \ \beta^L\ (Satisfactory\ Level)\right]?$$

4.3 Bi-Level Banking Risk Management Problem

Now the solution of the manager and follower are disclosed. However, two solutions are usually different because of nature between two levels objective functions. The manager knows that using the optimal decisions x_1^H as a control factors for the follower are not practical. It is more reasonable to have some tolerance that gives the follower an extent feasible region to search for his/her optimal solution, and reduce searching time or interactions.

In this way, the range of decision variable x_1 should be around x_1^H with maximum tolerance t_1 and the following membership function specify x_1 as:

$$\mu\ (x_1) = \begin{cases} \dfrac{x_1 - (x_1^H - t_1)}{t_1} & x_1^H - t_1 \le x_1 \le x_1^H \\ \dfrac{(x_1^H + t_1) - x_1}{t_1} & x_1^H \le x_1 \le x_1^H + t_1 \end{cases} \tag{12}$$

where x_1^H is the most preferred solution; the $(x_1^H - t_1)$ and $(x_1^H + t_1)$ is the worst acceptable decision; and that satisfaction is linearly increasing with the interval of $[x_1^H - t_1, x_1]$ and linearly decreasing with $[x_1\ , x_1^H + t_1]$, and other decision are not acceptable.

First, the manager goals may reasonably consider $F_1 \ge F_1^H$ is absolutely acceptable and $F_1 < F_1' = F_1(x_1^L, x_2^L)$ is absolutely unacceptable, and that the preference with $[F_1', F_1^H]$ is linearly increasing. This due to the fact that the follower obtained the optimum at (x_1^L, x_2^L), which in turn provides the manager the objective function values F_1', makes any $F_1 < F_1' = F_1(x_1^L, x_2^L)$ unattractive in practice.

The following membership functions of the manager can be stated as:

$$\mu'[F_1(x)] = \begin{cases} 1 & if \quad F_1(x) > F_1^H \\ \dfrac{F_1(x) - F_1'}{F_1^H - F_1'} & if \quad F_1' \le F_1(x) \le F_1^H \\ 0 & if \quad F_1' \ge F_1(x). \end{cases} \tag{13}$$

Second, the follower goals may reasonably consider the $F_2 \ge F_2^L$ is absolutely acceptable and $F_2 < F_2' = F_2(x_1^H, x_2^H)$ is absolutely unacceptable, and that the preference with $[F_2', F_2^L]$ is linearly increasing. In this way, the follower has the following membership functions for his/her goal:

$$\mu''[F_2(x)] = \begin{cases} 1 & if \quad F_2(x) > F_2^L \\ \dfrac{F_2(x) - F_2'}{F_2^L - F_2'} & if \quad F_2' \le F_2(x) \le F_2^L \\ 0 & if \quad F_2' \ge F_2(x). \end{cases} \tag{14}$$

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all decision - makers, we can solve the following mixed integer Tchebycheff problem.

$$Max\,\delta \tag{15}$$

Subject to

$$\frac{[(x_1^H + t_1) - x_1]}{t_1} \ge \delta I,$$

$$\frac{[x_1 - (x_1^H - t_1)]}{t_1} \ge \delta I,$$

$$\mu'[F_1(x)] \ge \delta,$$

$$\mu''[F_2(x)] \ge \delta,$$

$$(x_1, x_2) \in G,$$

$$t_1 > 0,$$

$$\delta \in [0,1].$$

Where δ is the overall satisfaction, and I is the column vector with all elements equal to one. Equation (15) is actually a fuzzy problem by Sakawa [10].

By solving problem (15). If the manager is satisfied with the solution then optimal solution is reached. Otherwise, he/she should provide new membership function for the control variable and objectives to the LLDM, until an optimal solution is reached. It is easy to see that there is an inverse correlation between t_1 and δ.

Remark 1: For solving this problem, the winqsb package is suggested as a basic solution tool.

5. Numerical Example for Problem

To demonstrate the solution method for risk banking management problem, let us consider the following example:

[Manager Level]

$$\underset{x_1}{Max}\ F_1(x_1, x_2) = \underset{x_1}{Max}\left[x_1{}^2 + x_2{}^2\right]$$

where x_2 solves

[Follower Level]

$$Max\ F_2(x_1, x_2) = Max\left[(x_1 - 1)^2 + x_2{}^2\right]$$

Subject to

$$(x_1, x_2) \in G = \{(x_1, x_2) \mid 2x_1 + x_2 \le 8,$$
$$x_1 + 2x_2 \le 6,$$
$$x_1, x_2 \ge 0.\}$$

First, the manager solves his / her problem as follows:

1- Find individual optimal solution by solving (4) – (5), we get: $(F_1^*, \overline{F_1}) = (16{,}0)$
2- By using (6), the manager build the membership function $\mu(F_1(x))$ then solve the mixed –integer Tchebycheff problem as follows:

Max λ,

subject to

$$(x_1, x_2) \in G$$
$$x^2{}_1 + x_2^2 - 16\lambda \ge 0\ .$$
$$\lambda \in [0\ ,1].$$

Whose solution is

$$(x_1^H\ ,\ x_2^H) = (4, 0), F_1^H = 16\ , \ and\ \ \lambda^H = 1\ .$$

Second, the follower solves his / her problem as follows:

1- Finds individual optimal solutions by solving (8) – (9), we get $(F_2^*, \overline{F_2}) = (10{,}0)$

2- By using (10) , the LLDM build membership function $\mu(F_2(x))$, then solve (11) as follows :

$Max\ \beta$,

subject to

$$(x_1\ ,\ x_2) \in G\,,$$
$$(x_1 - 1)^2 + x_2^2\ - 10\ \beta\ \geq 0\ ,$$
$$\lambda \in [0,1]\ .$$

Whose solution is

$$(x_1^L\ , x_1^L) = (2,2)\ ,\ F_2 = 5, and\ \beta^L\ = 0.49999.$$

Finally,

1- We assume the manager control decision ($x_1^H = 4$) with the tolerance 1.
2- By using (12) – (14), the follower solves the following mixed – integer Tchebycheff problem as follows:

$Max\,\delta$,

subject to

$$(x_1\ ,\ x_2\ )\ \in\ G\ ,$$
$$x_1\ -\ \delta\ \geq 3\ ,$$
$$x_1\ +\ \delta \leq\ 5\ ,$$
$$x_1^2\ + x_2^2\ - 12\,\delta \geq 4\ ,$$
$$(x_1\ - 1)^2\ + x_2^2\ + 4\,\delta \geq 9,$$
$$\delta \in\ [0,1]\ .$$

Whose optimal solution is

$(x_1^0\ ,\ x_2^0) = (4\ ,0),\ (F_1^0\ , F_2^0\ ) = (16,9), and\ \delta\ = 1$, (overall satisfaction for both decision-maker's).

6. Summary and Concluding Remarks

The bi-level programming problem can be thought as a static version of the Stackelberg strategy, which is used leader-follower game in which a Stackelberg strategy is used by the leader, or the higher-level decision-maker (manager), given the rational reaction of the follower, or the lower-level decision-maker (follower). This paper proposed a two-planner model and a solution method for solving this problem. This method used the concept of tolerance membership function to develop a fuzzy Max-Min decision model for generating Pareto optimal solution for this problem; an illustrative numerical example was given to demonstrate the obtained results.

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