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Research Paper

Some Properties of Upper and Lower α -Continuous Intuitionistic Fuzzy Multifunctions

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Abstract: *In this paper we introduce some properties of upper and lower* α*-continuous intuitionistic fuzzy multifunction* [28] *from a topological space to an intuitionistic fuzzy topological space*.

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, Intuitionistic fuzzy multifunctions, Lower alpha-continuous, Upper alpha-continuous.

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1. Introduction

After the introduction of fuzzy sets by Zadeh [30] in 1965 and fuzzy topology by Chang [6] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [2,3,4] as a generalization of fuzzy sets. In the last 27 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [7] introduced the concept of intuitionistic fuzzy topological spaces as a generalization of fuzzy topological spaces. In 1999, Ozbakir and Coker [23] introduced the concept intuitionistic fuzzy multifunctions and studied their lower and upper intuitionistic fuzzy semi continuity from a topological space to an intuitionistic fuzzy topological space. Recently the authors [28] of this paper introduced the concepts upper and lower α-continuous intuitionistic fuzzy multifunctions. In the present paper we study some of the properties of lower and upper α-continuous intuitionistic fuzzy multifunctions.

2. Preliminaries

Throughout this paper $(X,\mathcal{T})$ and (Y,Γ) represents a topological space and an intuitionistic fuzzy topological space respectively.

Definition 2.1:A subset *A* of a topological space $(X,\mathcal{T})$ is called :

- **(a)** Semi open [12] if $A \subset Cl(Int(A))$..
- **(a)** Semi closed [12] if its complement is semi open.
- **(b)** α-open [19] if $A \subset Int(Cl(Int(A)))$.
- **(c)** α-closed [19] if its complement is α -open.
- **(b)** preopen [16] if $A \subset Int(Cl(A))$.
- **(d)** preclosed [16] if its complement is preopen.

Remark.2.1. [12, 16,19]: every open (resp. closed) set is α-open (resp. α- closed) and every α-open (resp. α- closed) set is semi open (resp. semi closed) and preopen (resp. preclosed) but the separate converses may not be true.

The family of all α-open (resp. α-closed, semi open, semi closed, preopen, preclosed) subsets of topological space $(X,\mathcal{T})$ is denoted by αO(X) (resp. αC(X), SO(X),SC(X), PO(X), PC(X)). The intersection of all α-closed (resp. semi closed) sets of X containing a set A of X is called the α-closure [13] (resp. semi closure) of A. It is denoted by αCl(A) (resp. sCl(A)). The union of all α-open (resp. semi open) sub sets of A of X is called the α-interior [13] (resp. semi interior) of A .It is denoted by α-Int(A) (resp. sInt(A)) . A subset A of X is α-closed (resp. semi closed) if and only if $A \supset Cl\left(Int(Cl(A))\right)(resp.\ A \supset Int(Cl(A)))$. A subset N of a topological space $(X,\mathcal{T})$ is called a α-neighborhood [13] of a point x of X if there exists a α-open set O of X such that $x \in O \subset N$. A is a α-open in X if and only if it is a α-neighborhood of each of its points. A mapping f from a topological space $(X,\mathcal{T})$ to another topological space $(X^*,\mathcal{T}^*)$ is said to be α-continuous [14, 15] if the inverse image of every open set of X* is α-open in X. Every continuous mapping is α-continuous but the converse may not be true [14]. A multifunction F from a topological space $(X,\mathcal{T})$ to another topological space $(X^*,\mathcal{T}^*)$ is said to be lower α-continuous [18] (resp. upper α-continuous[18] at a point $x_0 \in X$ if for every α-neighborhood U of x_0 and for any open set W of X^* such that $F(x_0) \cap W \neq \emptyset$. (resp. $F(x_0) \subset W$) there is a α-neighborhood U of x_0 such that $F(x) \cap W \neq \emptyset$. (resp. $F(x) \subset W$)every $x \in U$.

Definition 2.2 [2, 3, 4]: Let Y be a nonempty fixed set. An intuitionistic fuzzy set $\tilde{A}$ in Y is an object having the form

$$\tilde{A} = \{< \mu_A(y), \nu_A(y) >: y \in Y\}$$

here the functions $\mu_{\tilde{A}}(y): Y \to I$ and $\nu_{\tilde{A}}(y): Y \to I$ denotes the degree of membership (namely $\mu_{\tilde{A}}(y)$) and the degree of non membership (namely $\nu_{\tilde{A}}(y)$) of each element $y \in Y$ to the set $\tilde{A}$ respectively, and $0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1$ for each $y \in Y$.

Definition 2.3 [2, 3, 4]: Let *Y* be a nonempty set and the intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ be in the form $\tilde{A} = \{< \mu_{\tilde{A}}(y), \nu_{\tilde{A}}(y) >: y \in Y\}$, $\tilde{B} = \{< \mu_{\tilde{B}}(y), \nu_{\tilde{B}}(y) >: y \in Y\}$ and let $\{\tilde{A}_\alpha : \alpha \in \Lambda\}$ be an arbitrary family of intuitionistic fuzzy sets in Y. Then:

(a) $\tilde{A} \subseteq \tilde{B}$ *if* $\forall\, y \in Y\, [\mu_{\tilde{A}}(y) \le \mu_{\tilde{B}}(y)$ *and* $\nu_{\tilde{A}}(y) \ge \nu_{\tilde{B}}(y)]$;

(b) $\tilde{A} = \tilde{B}$ *if* $\tilde{A} \subseteq \tilde{B}$ *and* $\tilde{B} \subseteq \tilde{A}$;

(c) $\tilde{A}^C = \{< y, \nu_{\tilde{A}}(y), \mu_{\tilde{A}}(y): y \in Y\}$;

(d) $\tilde{0} = \{< y, 0, 1 >: y \in Y\}$ and $\tilde{1} = \{< y, 1, 0 >: y \in Y\}$;

(e) $\cap \tilde{A}_\alpha = \{< y, \wedge \mu_{\tilde{A}}(y), \vee \nu_{\tilde{A}}(y) >: y \in Y\}$

(f) $\cup \tilde{A}_\alpha = \{< y, \vee \mu_{\tilde{A}}(y), \wedge \nu_{\tilde{A}}(y) >: y \in Y\}$

Definition 2.4 [8]: Two Intuitionistic Fuzzy Sets $\tilde{A}$ and $\tilde{B}$ of Y are said to be quasi coincident $(\tilde{A} q \tilde{B}$ for short$)$ if $\exists\, y \in Y$ such that
$\mu_{\tilde{A}}(y) > \nu_{\tilde{B}}(y)$ Or $\nu_{\tilde{A}}(y) < \mu_{\tilde{B}}(y)$.

Lemma 2.1[8]: For any two intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ of Y, $\rceil(\tilde{A} q \tilde{B}) \Leftrightarrow \tilde{A} \subset \tilde{B}^C$.

Definition 2.5[7]: An intuitionistic fuzzy topology on a non empty set Y is a family Γ of intuitionistic fuzzy sets in Y which satisfy the following axioms:

(O_1). $\tilde{0}, \tilde{1} \in \Gamma$,

(O_2). $\tilde{A}_1 \cap \tilde{A}_2 \in \Gamma$ for any $\tilde{A}_1, \tilde{A}_2 \in \Gamma$

(O_3). $\cup \tilde{A}_\alpha$ for any arbitrary family $\{\tilde{A}_\alpha : \alpha \in \Lambda\} \in \Gamma$,

In this case the pair (Y, Γ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in Γ is known as an intuitionistic fuzzy open set in *Y*. The complement $\tilde{B}^C$ of an intuitionistic fuzzy open set $\tilde{B}$ is called an intuitionistic fuzzy closed set in *Y*.

Definition 2.6 [7**]:** Let (Y, Γ) be an intuitionistic fuzzy topological space and $\tilde{A}$ be an intuitionistic fuzzy set in Y. Then the interior and closure of $\tilde{A}$ are defined by:

$Cl(\tilde{A}) = \cap \{\tilde{K} : \tilde{K}$ is an intuitionistic fuzzy closed set in Y and $\tilde{A} \subseteq \tilde{K}\}$,

$Int(\tilde{A}) = \cup \{\tilde{K} : \tilde{K}$ is an intuitionistic fuzzy open set in Y and $\tilde{K} \subseteq \tilde{A}\}$.

Definition 2.7[10]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (Y, Γ) is called :

(a) Intuitionistic fuzzy $\alpha -$ open if $\tilde{A} \subset \mathrm{Int}\,(\mathrm{Cl}\,(\mathrm{Int}(\tilde{A})))$,

(b) Intuitionistic fuzzy $\alpha - closed$ if its complement $\tilde{A}^C$ is Intuitionistic fuzzy $\alpha -$ open.

Remark.2.2.[10]: every Intuitionistic fuzzy open (resp. Intuitionistic fuzzy closed) set is Intuitionistic fuzzy $\alpha -$ open (resp.Intuitionistic fuzzy $\alpha -$ closed) but the separate converses may not be true.

Definition 2.8[13]: The intersection of all intuitionistic fuzzy $\alpha-$closed sets of Y containing $\tilde{A}$ is called the $\alpha-$closure [13] of $\tilde{A}$, it is denoted by $\alpha Cl(\tilde{A})$,

Definition 2.9 [23]: Let X and Y are two non empty sets. A function $F:(X,\mathcal{T})\to(Y,\Gamma)$ is called intuitionistic fuzzy multifunction if F(x) is an intuitionistic fuzzy set in Y, $\forall\, x\in X$.

Definition 2.10 [27]: Let $F:(X,\mathcal{T})\to(Y,\Gamma)$ is an intuitionistic fuzzy multifunction and A be a subset of X. Then $F(A)=\cup_{x\in A}F(x)$.

Definition 2.11[23]: Let $F:(X,\mathcal{T})\to(Y,\Gamma)$ be an intuitionistic fuzzy multifunction. Then the upper inverse $F^+(\tilde{A})$ and lower inverse $F^-(\tilde{A})$ of an intuitionistic fuzzy set $\tilde{A}$ in Y are defined as follows:

$F^+(\tilde{A})=\{x\in X: F(x)\subseteq\tilde{A}\}$ and

$F^-(\tilde{A})=\{x\in X: F(x)q\tilde{A}\}$.

Definition 2.12: An Intuitionistic fuzzy multifunction $F:(X,\mathcal{T})\to(Y,\Gamma)$ is said to be:

(a) Intuitionistic fuzzy upper semi continuous [23] (resp. upper $\alpha-$continuous [28], upper precontinuous [29], upper quasi-continuous [30]) at a point $x_0\in X$, if for any intuitionistic fuzzy open set $\tilde{W}\subset Y$ such that $F(x_0)\subset\tilde{W}$ there exists an open set $U\subset X\ (resp. U\in\alpha O(X), U\in PO(X), U\in SO(X))$ containing x_0 such that $F(U)\subset\tilde{W}$.

(b) Intuitionistic fuzzy lower semi continuous [23] (resp. lower $\alpha-$continuous [28], lower precontinuous [29], lower quasi-continuous[30]) at a point $x_0\in X$, if for any intuitionistic fuzzy open set $\tilde{W}\subset Y$ such that $F(x_0)q\tilde{W}$ there exists an open set $U\subset X\ (resp. U\in\alpha O(X), U\in PO(X), U\in SO(X))$ containing x_0 such that $F(U)q\tilde{W}, \forall x\in U$. Intuitionistic fuzzy upper semi-continuous (resp. upper $\alpha-$continuous, upper precontinuous, upper quasi-continuous) and intuitionistic fuzzy lower semi-continuous (resp. lower $\alpha-$continuous, lower precontinuous, lower quasi-continuous) if it is intuitionistic fuzzy upper semi-continuous (resp. upper $\alpha-$continuous, upper precontinuous, upper quasi-continuous) and Intuitionistic fuzzy lower semi-continuous (resp. lower $\alpha-$continuous, lower precontinuous, lower quasi-continuous) at each point of X.

3. Properties of Upper and Lower α-Continuous Intuitionistic Fuzzy Multifunctions

Theorem 3.1: An intuitionistic fuzzy multifunction $F:(X,\mathcal{T})\to(Y,\Gamma)$ is intuitionistic fuzzy upper $\alpha-$continuous if and only if it is intuitionistic fuzzy upper quasi-continuous and intuitionistic fuzzy upper pre-continuous.

Proof: Obvious from Theorem3.1 [27] and Lemma 3.1[21].

Corollary 3.1[26]: A fuzzy multifunction $F:(X,\mathcal{T})\to(Y,\sigma)$ is fuzzy upper $\alpha-$continuous if and only if it is fuzzy upper quasi-continuous and fuzzy upper pre-continuous.

Corollary 3.2[25]: A multifunction $F:(X,\mathcal{T})\to(Y,\theta)$ is upper α −continuous if and only if it is upper quasi-continuous and upper pre-continuous.

Theorem 3.2: An intuitionistic fuzzy multifunction $F:(X,\mathcal{T})\to(Y,\Gamma)$ is intuitionistic fuzzy lower α −continuous if and only if it is intuitionistic fuzzy lower quasi-continuous and intuitionistic fuzzy lower pre-continuous.

Proof: Obvious from Theorem 4.1 [27] Lemma 3.1 [21].

Corollary 3.3[26]: A fuzzy multifunction $F:(X,\mathcal{T})\to(Y,\sigma)$ is fuzzy lower α −continuous if and only if it is fuzzy lower quasi-continuous and fuzzy lower pre-continuous.

Corollary 3.4[25]: A multifunction $F:(X,\mathcal{T})\to(Y,\theta)$ is lower α −continuous if and only if it is lower quasi-continuous and lower pre-continuous.

Lemma 3.1. Let $F:(X,\mathcal{T})\to(Y,\Gamma)$ be an intuitionistic fuzzy multifunction. Then $[\alpha Cl(F)]^{-}(\tilde{V})=F^{-}(\tilde{V})$ for each intuitionistic fuzzy open set $\tilde{V}$ of Y.

Proof: Suppose that $\tilde{V}$ be any intuitionistic fuzzy open set of Y. Let $x\in[\alpha Cl(F)]^{-}(\tilde{V})$. If $x\notin F^{-}(\tilde{V})$ Then, $\urcorner(F(x)q\tilde{V})$. Which implies $F(x)\subset\tilde{V}^{c}$. Since $\tilde{V}^{c}$ is intuitionistic fuzzy closed and hence intuitionistic fuzzy α −closed ,$\alpha Cl(F(x))\subset\tilde{V}^{c}$. Consequently $x\notin[\alpha Cl(F)]^{-}(\tilde{V})$, which is a contradiction. Hence, $x\in F^{-}(\tilde{V})$. This shows that $[\alpha Cl(F)]^{-}(\tilde{V})\subset F^{-}(\tilde{V})$. Conversely, let $x\in F^{-}(\tilde{V})$. Then $F(x)q\tilde{V}$. Suppose that $x\notin[\alpha Cl(F)]^{-}(\tilde{V})$ Then $\urcorner(\alpha Cl(F(x)))q\tilde{V}$. And so $\alpha Cl(F(x))\subset\tilde{V}^{c}$. which implies that $F(x)\subset\tilde{V}^{c}$. Hence, $\urcorner(F(x)q\tilde{V})$. Which is a contradiction. Hence $x\in[\alpha Cl(F)]^{-}(\tilde{V})$ This shows that$[\alpha Cl(F)]^{-}(\tilde{V})\supset F^{-}(\tilde{V})$. Consequently, we obtain, $[\alpha Cl(F)]^{-}(\tilde{V})=F^{-}(\tilde{V})$.

Theorem 3.3: An intuitionistic fuzzy multifunction $F:(X,\mathcal{T})\to(Y,\Gamma)$ is intuitionistic fuzzy lower α-continuous if and only if $\alpha\mathrm{Cl}(F):(X,\mathcal{T})\to(Y,\Gamma)$ is intuitionistic fuzzy lower α-continuous.

Proof: ***Necessity.*** Suppose that F is intuitionistic fuzzy lower α-continuous. Let $x\in X$ and $\tilde{V}$ be any intuitionistic fuzzy open set of Y such that $\alpha\mathrm{Cl}(F(x))q\tilde{V}$. By Lemma 3.1, we have $x\in[\alpha Cl(F)]^{-}(\tilde{V})=F^{-}(\tilde{V})$. Since F is intuitionistic fuzzy lower α-continuous, there exists $U\in\alpha\mathrm{O}(\mathrm{X})$ containing *x* such that $F(u)q\tilde{V},\forall u\in U$. Now $\tilde{V}$ be an intuitionistic fuzzy open set of Y. $F(u)q\tilde{V}\Longrightarrow\alpha\mathrm{Cl}(F(x))q\tilde{V}$. This shows that $\alpha\mathrm{Cl}(F)$ is intuitionistic fuzzy lower α-continuous.

Sufficiency. Suppose that $\alpha\mathrm{Cl}(F):(X,\mathcal{T})\to(Y,\Gamma)$ is intuitionistic fuzzy lower α-continuous. Let $x\in X$ and V be any α-open set of Y such that $F(x)q\tilde{V}$. By Lemma 3.1 we have $x\in[\alpha Cl(F)]^{-}(\tilde{V})=F^{-}(\tilde{V})$. and hence $\alpha\mathrm{Cl}(F(x))q\tilde{V}$. Since, $\alpha\mathrm{Cl}(\mathrm{F})$ is intuitionistic fuzzy lower α-continuous, there exists $U\in\alpha(\mathrm{X})$ containing *x* such that hence, $\alpha\mathrm{Cl}(F(x))q\tilde{V},\forall u\in U$. Hence, $F(u)q\tilde{V},\forall u\in U$ and F is intuitionistic fuzzy lower α-continuous.

Corollary 3.5[26]: A fuzzy multifunction $F:(X,\mathcal{T}) \to (Y,\sigma)$ is fuzzy lower α-continuous if and only if $\alpha\mathrm{Cl}(F):(X,\mathcal{T}) \to (Y,\sigma)$is fuzzy lower α-continuous.

Corollary 3.6[25]: A fuzzy multifunction $F:(X,\mathcal{T}) \to (Y,\theta)$ is fuzzy lower α-continuous if and only if $\alpha\mathrm{Cl}(F):(X,\mathcal{T}) \to (Y,\theta)$is fuzzy lower α-continuous.

Lemma 3.2: Let A and B be subsets of a topological space $(X,\mathcal{T})$.

(a) If $A \in SO(X) \cup PO(X)$ and $B \in \alpha O(X)$, then $A \cap B \in \alpha O(A)$.

(b) If $A \subset B \subset X, A \in \alpha O(B)$ and $B \in \alpha O(X)$, then $A \in \alpha O(X)$

Theorem 3.4: If an intuitionistic fuzzy multifunction $F:(X,\mathcal{T}) \to (Y,\Gamma)$ is intuitionistic fuzzy upper α-continuous (resp. intuitionistic fuzzy lower α-continuous) and $X_0 \in PO(X) \cup SO(X)$ then the restriction $F|X_0 : X_0 \to Y$ is intuitionistic fuzzy upper α-continuous (resp. intuitionistic fuzzy lower α-continuous).

Proof: We prove only the assertion for F intuitionistic fuzzy upper α-continuous, the proof for F intuitionistic fuzzy lower α-continuous being analogous. Let $x \in X$ and $\tilde{V}$ be any intuitionistic fuzzy open set of Y such that $F|X_0(x) \subset \tilde{V}$. Since F intuitionistic fuzzy upper α-continuous and $F|X_0(x) = \tilde{V}$, there exists $U \in \alpha(X)$ containing x such that $F(U) \subset \tilde{V}$. Set $U_0 = U \cap X_0$, then By Lemma 3.2 we have $x \in U_0 \in \alpha O(X_0)$ and $\left(F|X_0\right)(U_0) \subset \tilde{V}$. This shows that $F|X_0 : X_0 \to Y$ is intuitionistic fuzzy upper α-continuous.

Corollary 3.7[26]: If a fuzzy multifunction $F:(X,\mathcal{T}) \to (Y,\sigma)$ is fuzzy upper α-continuous (resp. fuzzy lower α-continuous) and $X_0 \in PO(X) \cup SO(X)$ then the restriction $F|X_0 : X_0 \to Y$ is fuzzy upper α-continuous (resp. fuzzy lower α-continuous).

Corollary 3.8 [25]: If a multifunction $F:(X,\mathcal{T}) \to (Y,\theta)$ is upper α-continuous (resp. lower α-continuous) and $X_0 \in PO(X) \cup SO(X)$ then the restriction $F|X_0 : X_0 \to Y$ is upper α-continuous (resp. lower α-continuous).

Theorem 3.5: An intuitionistic fuzzy multifunction $F:(X,\mathcal{T}) \to (Y,\Gamma)$ is intuitionistic fuzzy upper α-continuous (resp. intuitionistic fuzzy lower α-continuous) if for each $x \in X$ there exists $X_0 \in \alpha O(X)$ containing x such that the restriction $F|X_0 : X_0 \to Y$ is intuitionistic fuzzy upper α-continuous (resp. intuitionistic fuzzy lower α-continuous).

Proof: We prove only the assertion for F intuitionistic fuzzy upper α-continuous, the proof for F intuitionistic fuzzy lower α-continuous being analogous. Let $x \in X$ and $\tilde{V}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{V}$. There exists $X_0 \in \alpha O(X)$ containing x such that $F|X_0(x) = F(x)$ is intuitionistic fuzzy upper α-continuous, there exists $U_0 \in \alpha O(X_0)$ containing x such that $F|X_0(U_0) \subset \tilde{V}$. Then By Lemma 3.2 we have $x \in U_0 \in \alpha O(X)$ and $F(u) = F|X_0(u)$ for $u \in U_0$.This shows that $F:(X,\mathcal{T}) \to (Y,\Gamma)$ is intuitionistic fuzzy upper α-continuous.

Corollary 3.9[26]: A fuzzy multifunction $F:(X,\mathcal{T}) \rightarrow (Y,\sigma)$ is fuzzy upper α-continuous (resp. Fuzzy lower α-continuous) if for each $x \in X$ there exists $X_0 \in \alpha O(X)$ containing x such that the restriction $F|X_0: X_0 \rightarrow Y$ is fuzzy upper α-continuous (resp. fuzzy lower α-continuous).

Corollary 3.10 [25]: A multifunction $F:(X,\mathcal{T}) \rightarrow (Y,\theta)$ is upper α-continuous (resp. lower α-continuous) if for each $x \in X$ there exists $X_0 \in \alpha O(X)$ containing x such that the restriction $F|X_0: X_0 \rightarrow Y$ is upper α-continuous (resp. lower α-continuous).

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