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Research Paper

On Bi-Level Multi-Objective Large Scale Quadratic Programming Problem

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Abstract: *This paper presents a bi level large scale multi objective quadratic programming problem (BLLSMOQPP) with fuzzy parameters in the objective functions problem. In the first phase convert the fuzzy parameters of the problem to equivalent crisp problem to make it easy in solving. In the second phase use Taylor series to convert the quadratic problem to linear problem to be easy for solving with the decomposition algorithm that deals with large scale constraint. In addition, a numerical example is provided to demonstrate the correctness of the proposed solution.*

Keywords: Bi-level, multi-objective, large scale, quadratic programming, fuzzy numbers.

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1. Introduction

Bi-level programming (BLP) problem has a subset of their variables constrained to be an optimal solution of another problem parameterized by the remaining variables. They have been applied to decentralize planning problems involving a decision process with a hierarchical structure. The basic connect of the BLP technique is that a first level decision maker (FLDM) sets his goals and/or decisions and then asks each subordinate level of the organization for their optima which are calculated in isolation; the second level decision maker (SLDM) decision is then submitted and modified by the FLDM with consideration of the overall benefit for the organization; and the process continued until an optimal solution is reached [15].

There are many familiar structures for large scale optimization problems such as: the block angular structure, angular and dual- angular structure to the constraints, and several kinds of decomposition methods for linear and nonlinear programming problems with those structures have been proposed in [1, 2, 3, 4, 5, 6].

Emam presented a bi-level integer non-linear programming problem with linear or non-linear constraints [11] and proposed an interactive approach to solve a bi-level integer multi-objective fractional programming problem in [10]. Baky [7] introduced two new algorithms to solve multi-level multi-objective linear programming problems through the fuzzy goal programming approach. The membership functions for the defined fuzzy goals of all objective functions at all levels were developed. Then the fuzzy goal programming approach was used to obtain the satisfactory solution for all decision makers.

Abo–Sinna and Abou-El-Enien [5] extend TOPSIS for solving interactive large scale multiple Objective programming problems involving fuzzy parameters. These fuzzy parameters are characterized as fuzzy numbers. For such problems, the α-Pareto optimality is introduced by extending the ordinary Pareto optimality on the basis of the α-Level sets of fuzzy numbers. An interactive fuzzy decision making algorithm for generating α-Pareto optimal solution through TOPSIS approach is provided where the decision maker (DM) is asked to specify the degree α and the relative importance of objectives.

Osman [13] presented a method for solving a special class of large scale fuzzy multi-objective integer problems depending on the decomposition algorithm

This paper is organized as follows: Section 2 presents a problem formulation and solution of a bi-level multi-objective large scale quadratic programming problem with fuzzy parameters in the objective functions. Section 3 converts the fuzzy number in the objective functions into deterministic functions. Section 4 presents a Taylor series approach for bi-level large scale multi-objective quadratic programming problem (BLLSMOQPP) then converts the quadratic objective functions. In section 5, the decomposition method for large scale bi-level linear programming problem is presented. In Section 6, an example is provided to describe the developed results. Finally, Section 7 concludes the paper states some open points for future research work in the area of fuzzy bi-level multi-objective quadratic programming optimization problems.

2. Problem Formulation and Solution Concept

The bi-level large scale multi-objective quadratic programming problem with fuzzy numbers in the objective functions may be formulated as follows:

[FLDM]

$$\underset{x_1,x_2}{Max}\; F_1(x,\tilde{u}) = \underset{x_1,x_2}{Max}\;[\,f_{11}(x,\tilde{u}_1), f_{12}(x,\tilde{u}_2),\ldots, f_{1n1}(x,\tilde{u}_{1n_1})] \tag{1}$$

Where x_3, x_4 solves

[SLDM]

$$\underset{x_3,x_4}{Max}\; F_2(x,\tilde{u}) = \underset{x_3,x_4}{Max}\;[\,f_{21}(x,\tilde{u}_1), f_{22}(x,\tilde{u}_2),\ldots, f_{2n2}(x,\tilde{u}_{1n_1})] \tag{2}$$

Subject to

$$x \in G. \tag{3}$$

Where

$$G = \{ a_{01}x_1 + a_{02}x_2 + a_{0m}x_m \le b_0,\ d_1x_1 \le b_1,\ d_2x_2 \le b_2,\ d_mx_m \le b_m,\ x_1,...,x_m \ge 0 \}.$$

Where F_1 and F_2 are nonlinear functions contain fuzzy numbers defined on R^n.

Let $x_1, x_2\ x_3, x_4$ be a real vector variables indicating the first decision level's choice and the second decision level's choice. Moreover, the FLDM has x_1, x_2 indicating the first decision level choice, the SLDM have x_3, x_4 indicating the second decision level choice.

In the above problem (1) – (3), $x_j \in R\ ,(j = 1,2,...,m)$ be a real vector variables, an $\tilde{u}_i$ n-dimensional row vector of fuzzy parameters in the objective functions, G is the large scale linear constraint set where, $b = (b_0,...,b_m)^T$ is $(m+1)$ vector, and $a_{01},...,a_{0m}, d_1,...,d_m$ are constants.

Definition 2.1: *For any* $(x_1, x_2 \in G_1 = \{x_1, x_2 | (x_1,.x_2, x_3,..., x_m) \in G\})$ *given by FLDM, if the decision-making variable* $(x_3, x_4 \in G_2 = \{x_3, x_4 |\ (x_1,.x_2, x_3,..., x_m) \in G\})$ *is the optimal solution of the SLDM, then* $(x_1,.x_2, x_3, x_4)$ *is a feasible solution of (BLLSMOQPP).*

Definition 2.2: *If* $x^* \in R^m$ *is a feasible solution of the (BLLSMOQPP); no other feasible solution* $x \in G$ *exists, such that* $F_1(x^*) \le F_1(x)$ *so* x^* *is the optimal solution of the (BLLSMOQPP).*

3. Ranking Functions

To solve bi-level multi-objective large scale quadratic programming problem with fuzzy numbers in the objective functions using linear ranking technique that convert fuzzy number form into equivalent crisp form by using [14]**:**

Definition 3.1 [13]: *If* $\tilde{A} = (a,b,c,d) \in F(R)$*, then a linear ranking function is defined as*

$$\Re(\tilde{A}) = a + b + \frac{1}{2}(d - c). \tag{4}$$

Definition 3.2[13]: *Let* $\tilde{A}\ (a_1, b_1,\ c_1,\ d_1)$ *and* $\tilde{B}\ (a_2, b_2,\ c_2,\ d_2)$ *is two trapezoidal fuzzy numbers and* $x \in R$. *A convenient method for comparing of the fuzzy numbers is by using of ranking functions. A ranking function is a map from* $F(R)$ *into the real line. So, the orders on F(R) as follows:*

1. $\tilde{A} \ge \tilde{B}$ If and only if $\Re(\tilde{A}) \ge \Re(\tilde{B})$.
2. $\tilde{A} > \tilde{B}$ If and only if $\Re(\tilde{A}) > \Re(\tilde{B})$.
3. $\tilde{A} = \tilde{B}$ If and only if $\Re(\tilde{A}) = \Re(\tilde{B})$.

Where $\tilde{A}$ and $\tilde{B}$ are in $F(R)$.

Then the problem can be understood as the corresponding deterministic bi-level large scale multi-objective quadratic programming problem in the objective function as following:

[FLDM]

$$\underset{x_1, x_2}{Max} \quad f_1(x) = [\, f_{11}(x), f_{12}(x), \ldots, f_{1n1}(x) \,] \tag{5}$$

Where x_3, x_4 solves

[SLDM]

$$\underset{x_3, x_4}{Max} \quad f_2(x) = [\, f_{21}(x), f_{22}(x), \ldots, f_{2n2}(x) \,] \tag{6}$$

Subject to

$$x \in G. \tag{7}$$

Where

$$G = \{\, a_{01}x_1 + a_{02}x_2 + a_{0m}x_m \le b_0,\ d_1x_1 \le b_1,\ d_2x_2 \le b_2,\ d_mx_m \le b_m,\ x_1,\ldots,x_m \ge 0. \}$$

4. Taylor Series Approach

This problem is bi level large scale multi-objective quadratic programming problem (BLLSMOQPP) and it's difficult to solve this problem using a decomposition algorithms, so we transform the objective functions by using 1^{st} order Taylor series polynomial in the following form[16].

$$K_i(x) \cong F_i^{\wedge}(x) = F_i(x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}^*) \frac{\partial F_i(x_j^*)}{dx_j}, (j = 1,2,\ldots m), (i = 1,2) \tag{8}$$

Then we use the weighting method to transform the objective functions in the upper level and lower level from multi-objective into single objective .the weight of the first objective is greater than the weight of the second objective so the BILSPP can be written as:

[FLDM]

$$\underset{x_1, x_2}{Max} \quad Q_1(x) \tag{9}$$

Where x_3, x_4 solves

[SLDM]

$$\underset{x_3, x_4}{Max} \quad Q_2(x) \tag{10}$$

Subject to

$$x \in G. \tag{11}$$

5. Decomposition Algorithm for Bi Level Large Scale Linear Programming Problem

The bi level large scale linear programming problem is solved by adopting the leader-follower Stackelberg strategy combine with Dantzig and Wolf decomposition method [9]. First, the optimal solution that is acceptable to the FLDM is obtained using the decomposition method to break the large scale problem into n-sub problems that can be solved directly.

The decomposition technique depends on representing the BLLSLPP in terms of the extreme points of the sets $d_j x_j \le b_j$, $x_j \ge 0$, $j = 1,2,..,m$ .To do so, the solution space described by each $d_j x_j \le b_j$, $x_j \ge 0$, $j = 1,2,..,m$ must be bounded and closed.

After that by inserting the upper level decision variable to the lower level for him/her to search for the optimal solution using Dantzig and Wolf decomposition method [9], then the decomposition method break the large scale problem into n-sub problems that can be solved directly and obtain the optimal solution for his/her problem which is the optimal solution to the BILSPP.

Theorem 1: *The decomposition algorithm terminates in a finite number of iterations, yielding a solution of the large scale problem.*

To prove theorem 1 above, the reader is referred to [9].

5.1. The First-Level Decision-Maker (FLDM) Problem

The FLDM problem of the BLLSLPP is as follows:

[FLDM]

$$Max\ F_1(x) = Max \sum_{j=1}^{m} c_{1j} x_j \tag{12}$$

Subject to

$x \in G$.

To obtain the optimal solution of the first level; suppose that the extreme points of $d_j x_j \le b_j$, $x_j \ge 0$ are defined as $\hat{x}_{jk}$, $k = 1,2$ where x_j defined by:

$$x_j = \sum_{k=1}^{k_j} \beta_{jk} \hat{x}_{jk} \quad , \ j = 1,...,m. \tag{13}$$

, and $\beta_{jk} \ge 0$, for all k and $\sum_{k=1}^{k_j} \beta_{jk} = 1$.

Now, the first level problem in terms of the extreme points to obtain the following master problem of the first Level are formulated as stated in [9]:

$$\text{Max} \sum_{k=1}^{k_1} c_{11} \hat{x}_{1k} \beta_{1k} + \sum_{k=1}^{k_2} c_{12} \hat{x}_{2k} \beta_{2k} + ... + \sum_{k=1}^{k_n} c_{1n} \hat{x}_{nk} \beta_{nk}, \tag{14}$$

Subject to

$$\sum_{k=1}^{k_1} a_{01}\hat{x}_{1k}\beta_{1k}+\sum_{k=1}^{k_2} a_{02}\hat{x}_{2k}\beta_{2k}+\ldots+\sum_{k=1}^{k_n} a_{0n}\hat{x}_{nk}\beta_{nk}\le b_0, \tag{15}$$

$$\sum_{k=1}^{k_1}\beta_{1k}=1,$$

$$\sum_{k=1}^{k_2}\beta_{2k}=1,$$

$$\sum_{k=1}^{k_n}\beta_{nk}=1,$$

$\beta_{jk}\ge 0$, for all j and k.

The new variables in the first level problem are β_{jk} which determined using Balinski's algorithm [8]. Once their optimal values β^{*}_{jk} are obtained, then the optimal solution to the original problem can be found by back substitution as follow:

$$x_j=\sum_{k=1}^{k_j}\overset{*}{\beta}_{jk}\hat{x}_{jk}\ ,\ j=1,2 \tag{16}$$

It may appear that the solution of the upper level problem requires prior determination of all extreme points $\hat{x}_{jk}$.

To solve the first level problem by the revised simplex method, it must determine the entering and leaving variables at each iteration. Let us start first with the entering variables.

Given C_B and B^{-1} of the current basis of the First Level problem, then for non-basic β_{jk} :

$$z_{jk}-c_{jk}=C_B B^{-1}P_{jk}-c_{jk} \tag{17}$$

Where

$$c_{jk}=c_j\hat{x}_{jk} \text{ And } P_{jk}=\begin{bmatrix} a_j\hat{x}_{jk}\\ 1\\ 0 \end{bmatrix} \tag{18}$$

Now, to decide which of the variables β_{jk} should enter the solution it must determine:

$$\overset{*}{z}_{jk}-\overset{*}{c}_{jk}=\min\{z_{jk}-c_{jk}\} \tag{19}$$

Consequently, if $\overset{*}{z}_{jk}-\overset{*}{c}_{jk}\le 0$, then according to the maximization optimality condition, $\overset{*}{\beta}_{jk}$ must enter the solution; otherwise, the optimal has been reached.

5.2. The Second-Level Decision-Maker (SLDM) Problem

Finally, according to the mechanism of the BLLSLPP, the First Level variables x_1^F, x_2^F should be passed to the Second-Level; so the second -level problem can be written as follows:

$$Max F_2(x) = Max \sum_{j=1}^{m} c_{2j} x_j, \tag{20}$$

Subject to

$$(x_1^F, x_2^F, ..., x_m) \in G. \tag{21}$$

To obtain the optimal solution of the second -level problem; the SLDM solves his master problem by the decomposition method [9] as the first level.

Now the optimal solution $(x_1^F, x_2^F, x_3^S, x_4^S)$ of the second -Level is the optimal solution of the BLLSLPP.

6. An Algorithm for Solving (BLLSMOQPP) with Fuzzy Numbers

Step 1: Compute $\Re(\tilde{A})$ for all the coefficients of the problem (1) – (3), where $\tilde{A}$ is trapezoidal fuzzy number.

Step 2: Convert from fuzzy to crisp formula.

Step 3: Formulate the equivalent bi-level large scale multi-objective quadratic programming.

Step 4: Convert bi level large scale multi-objective quadratic programming to linear by using Taylor series approach as follow:

$$H_i(x) \cong \hat{F_i}(x) = F_i(x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}^*) \frac{\partial F_i(x_i^*)}{dx_j}, (j = 1,2,...n)$$

Step 5: Use weight method to convert from multi objective to single objective.

Step 6: Start with the first level problem and convert the master problem in terms of extreme points of the sets $d_j x_j \le b_j, x_j \ge 0$, $j = 1,2,3$.

Step 7: Determine the extreme points $x_j = \sum_{k=1}^{k_j} \beta_{jk} \hat{x}_{jk}, j = 1,2,3$ using Balinski's algorithm [14]

Step 8: Set $k = 1$.

Step 9: Compute $z_{jk} - c_{jk} = C_B B^{-1} P_{jk} - c_{jk}$.

Step 10: If $\overset{*}{z}_{jk} - \overset{*}{c}_{jk} \le 0$, then go to Step 11; otherwise, the optimal solution has been reached, go to Step 16.

Step 11: Determine $\hat{X}_{jk}$ associated with $\min\{\overset{*}{z}_{jk} - \overset{*}{c}_{jk}\}$

Step 12: β_{jk} Associated with extreme point $\hat{X}_{jk}$ must enter the solution

Step 13: Determine the leaving variable

Step 14: The new basis is determined by replacing the vector associated with leaving variable with the vector β_{jk} .

Step 15: Set $k = k + 1$, go to step 9.

Step 16: If the SLDM obtain the optimal solution go to Step 19, otherwise go to Step 17

Step 17: Set $(x_1, x_2) = (x_1^F, x_2^F)$ to the SLDM constraints.

Step 18: The SLDM formulate his problem, go to Step 8.

Step 19: $(x_1^F, x_2^F, x_3^S, x_4^S)$ Is as an optimal solution for bi-level large scale linear programming problem, then stop.

7. Numerical Example

To demonstrate the solution for (BLLSMOQPP) with fuzzy numbers, let us consider the following problem:

[FLDM]

$$\underset{x_1,x_2}{Max}\ F_1(x_1,x_2) = \underset{x_1,x_2}{Max}[(1,2,4,2)x_1^2 + (3,1,1,1)x_2^2 + (2,1,3,1)x_3 \quad ,(5,3,2,2)x_1^2 + (2,1,5,3)x_2^2]$$

Where x_3, x_4 solves

[SLDM]

$$\underset{x_3,x_4}{Max} F_2(x_3,x_4) = \underset{x_1,x_2}{Max}[(3,5,2,2)x_1^2 + (2,1,3,1)x_3^2 + (1,1,1,1)x_4^2 \quad ,(4,1,5,3)x_3^2 + (6,1,4,2)x_4]$$

Subject to

$$x_1 + x_2 + x_3 + x_4 \le 80,$$
$$x_1 + 3x_2 \le 60,$$
$$4x_3 + 2x_4 \le 20,$$
$$x_3 + x_4 \le 12,$$

$$x_1, x_2, x_3, x_4 \ge 0.$$

Firstly, Applying ranking function R (A) = a + b + 1/2 (d - c) to transform the fuzzy number form in to equivalent crisp form so, the problem reduces to

[FLDM]

$$\underset{x_1,x_2}{Max}\, F_1(x_1,x_2)=\underset{x_1,x_2}{Max}[2x_1^2+4x_2^2+2x_3,8x_1^2+2x_2^2]$$

Where x_3, x_4 solves

[SLDM]

$$\underset{x_3,x_4}{Max} F_2(x_3,x_4)=\underset{x_1,x_2}{Max}[8x_1^2+2x_3^2+2x_4^2,4x_3^2+6x_4]$$

Subject to

$$x_1+x_2+x_3+x_4\le 80,$$
$$x_1+3x_2\le 60,$$
$$4x_3+2x_4\le 20,$$
$$x_3+x_4\le 12,$$
$$x_1,x_2,x_3,x_4\ge 0.$$

Secondly, applying the first order Taylor series to convert the quadratic objectives function to single objective and use the weighting method so the BLLSPP is written as following:

[FLDM]

$$\underset{x_1,x_2}{Max}\, F_1(x_1,x_2)=\underset{x_1,x_2}{Max} 10x_1+4x_2+x_3-17,$$

Where x_3, x_4 solves

[SLDM]

$$\underset{x_3,x_4}{Max} F_2(x_3,x_4)=\underset{x_3,x_4}{Max} 8x_1+5x_4-11,$$

Subject to

$$x_1+x_2+x_3+x_4\le 80,$$
$$x_1+3x_2\le 60,$$
$$4x_3+2x_4\le 20,$$
$$x_3+x_4\le 12,$$
$$x_1,x_2,x_3,x_4\ge 0.$$

The first level problem is formulated as:

$$\underset{x_1,x_2}{Max}\, F_1(x_1,x_2)=\underset{x_1,x_2}{Max} 10x_1+4x_2+x_3-17,$$

Subject to

$x\in G.$

After four iterations the first level decision maker optimal solution is obtained:

$\left(x_1^F, x_2^F, x_3^F, x_4^F\right) = (60,0,5,0).$

So, $F_1 = 588$

Then take the first level decision maker solution and set $\left(x_1^F, x_2^F\right) = (60,0)$ to the second level constraint.

The second level decision maker will repeat the same steps as the first level decision maker until the second level decision maker get the optimal solution so:

$\left(x_3^s, x_4^s\right) = (0,10).$

So,

$\left(x_1^s, x_2^s, x_3^s, x_4^s\right) = (60,0,0,10).$

$F^*_1 = 583, \; F^*_2 = 519$

8. Summary and Concluding Remarks

This paper presented a bi level large scale multi objective quadratic programming problem with fuzzy parameters in the objective functions problem in which the objective functions at every level are to be maximized. In the first phase we converted the fuzzy parameters of the problem to equivalent crisp problem to make it easy in solving. In the second phase we used Taylor series to convert the quadratic problem to linear problem to be easy for solving with the decomposition algorithm that deals with large scale constrain. Finally the numerical example to demonstrate the result of this paper was introduced.

However, there are many other aspects, which should by explored and studied in the area of a large scale bi-level optimization such as:

1. An algorithm for solving large scale multi-level integer fractional multi-objective decision-making problems with random parameters in the objective functions; in the constraints and with integrality conditions.
2. An algorithm for solving large scale multi-level integer fractional multi-objective decision-making problems with Rough parameters in the objective functions; in the constraints and with integrality conditions.
3. An algorithm for solving large scale multi-level integer quadratic multi-objective decision-making problems with fuzzy parameters in the objective functions; in the constraints and with integrality conditions.

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