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Research Paper

Reactive Poiseuille Flow of a Power-Law Fluid with Thermal Radiation through a Saturated Porous Medium

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Abstract: *In this work, effects of thermal radiation on Poiseuille flow of a reactive power-law fluid of grade three between heated parallel plates through a saturated porous medium is considered. It is assumed that the fluid reacts satisfying Arrhenius law. The chemical reaction is assumed to be strongly exothermic. We employed Galerkin weighted residual method to solve the resulting non-linear equations. The effects of various physical parameters on the flow system were reported graphically.*

Keywords: Power-law fluid, weighted residual method, thermal radiation, Third grade fluid and Poiseuille flow.

1.0 Introduction

The importance of third grade fluid cannot be overemphasized due to its applications in many engineering systems such as flow of moisture through porous industrial materials, packed-bed chemical reactors, preheating coal-water mixture, ceramic processing, catalytic reactors, polymer solution, molten plastics, oil recovery processes, to mention but just a few.

The second grade fluid model is able to predict the normal stress differences which are characteristic of non-Newtonian fluids. It does not predict the effect of shear thinning and thickening phenomena that many reveal. The third grade fluid models are capable of describing such phenomena. The differential equations that arise when modeling non-Newtonian incompressible fluid flows are highly non-linear and complicated. It is well known from conductive theory of thermal explosion that the temperature equation expresses the balance between heat generated and heat conduction in terms of standard dimensionless parameter. Physically, the $Frank-Kamenetskii$ parameter ψ is a reflection of the internal properties of a given system. In some recent researches the variation of thermal conductivity parameters and power-law fluid were considered [8, 9]. And the following are some of the recent works in this regard.

Yurusoy et al [1] studied entropy analysis for third grade fluid flow with Vogel's models of viscosity in annular pipe. Makinde [2] employed Hermite-Pade approximations to evaluate thermal radiation effect of inherent irreversibility in a variable viscosity channel flow. Jayeoba and Okoya [3] employed analytical approximations to determine the velocity and temperature fields for steady flow of a third grade fluid in a pipe. Rilvin and Ericksen [4] analyzed stress deformation relation for isotropic materials. Szeri and Rajagopal [5] examined the effects of variable viscosity parameter and viscous dissipation parameter on the flow of a Non-Newtonian fluid between heated parallel plates. Their results show that the temperature and velocity distribution remain sensibly invariant with respect to the variable viscosity parameter. Furthermore, Lazarus [6] studied the effects of variable viscosity on the flow velocity and temperature field using semi-implicit finite difference scheme of Laminar flow in a channel filled with saturated porous media. The results show that the velocity field and the fluid temperature increases as variable viscosity parameter increases. Haroon et al [7] examined analysis of poiseuille flow of a reactive power-law fluid between parallel plates. The results show that the shear thinning/thickening behavior depends on the power-law index and the pressure gradient. Hayat et al[8] examined the effect of Joule heating and thermal radiation in flow of third grade fluid over radiative surface. Also Singh[9] considered the effects of viscous dissipation and variable viscosity effects on MHD boundary layer flow in porous medium past a moving vertical plate with suction. Motivated by [7,8] we consider a reactive Power-law fluid and examined the effect of radiating energy.

2.0 Governing Equations

Following Szeri and Rajagopal [5], an incompressible, homogeneous fluid of third grade is characterized by Cauchy stress τ of the following form:

$$\begin{aligned}\tau = &- pI + \mu(T)A_1 + \alpha_1(T)A_2 + \alpha_2(T)A_1^2 \\ &+ \beta_1(T)A_3 + \beta_2(T)[A_1A_2 + A_2A_1] + \beta_3(T)(trA_1^2)A_1.\end{aligned} \tag{1}$$

where $-pI$ denote the indeterminate part of the stress due to the constraint of incompressibility $\mu(T)$ is the coefficient of viscosity and $\alpha_1(T), \alpha_2(T)$ are material moduli, usually referred to as normal stress coefficients. The kinematic tensors A_1, A_2 are defined by [4] through

$$A_1 = (grad\ v) + (grad\ v)^{\tau} \tag{2}$$

$$A_n = \frac{d}{dt}A_{n-1} + A_{n-1}(grad\ v) + (grad\ v)^T A_{n-1} \quad n = 2,3. \tag{3}$$

Here $\frac{d}{dt}$ denotes material time derivative and v is the velocity vector. The above model contains, as a special subclass, the classical linearly viscous model (the case when all the coefficients expect μ are set equal to zero).

The thermodynamics and stability of model (1) has been studied in detail [4]. The thermodynamic compatibility in the sense that all motions of the fluid meet the Clausius- Duhem inequality, which is generally interpreted as a statement of the second law of thermodynamics, and the assumption that the specific Helmholtz free energy of the fluid be a minimum when the fluid is in "equilibrium", places restrictions on the structure of the constitutive equations which model the fluid. It has been shown [5] that the response functions $\Psi, \tau\, and\ q$ for specific Helmholtz free energy, the stress and the heat flux, respectively, of an incompressible, homogeneous fluid of third grade are compatible with thermodynamics only if

(i) the viscosity $\mu(T)$ is non-negative, $\mu(T) \geq 0,$ (4)

(ii) the normal stress coefficients $\alpha_1(T) and\ \alpha_2(T)$ meet the requirements,

$$\alpha_1(T) \geq 0 - \sqrt{24\mu(T)\beta_3(T)} \leq \alpha_1(T) + \alpha_2(T) < \sqrt{24\mu(T)\beta_3(T)} \tag{5}$$

(iii) the material coefficients $\beta_1(T), \beta_2(T) and\ \beta_3(T)$
satisfy $\beta_1(T) = 0, \beta_2(T) = 0, \beta_3(T) \geq 0$ (6)

(iv) the specific Helmholtz free energy Ψ has the form

$$\Psi = \hat{\Psi}(T, L) = \hat{\Psi}(T, 0) + \frac{\alpha_1(T)}{4\rho}\left|L + L^T\right|^2 \tag{7}$$

In the above expressions

$$L = grad\, v, \tag{8}$$

ρ denotes the density and $|A|$ denotes the trace norm of A.

In our analysis we assume that the fluid is thermodynamically compatible; hence the stress constitutive relation (1) reduces to

$$\tau = -\,pI + \mu(T)A_1 + \alpha_1(T)A_2 + \alpha_2(T)A_1^2 + \beta(T)\left(trA_1^2\right)A_1. \tag{9}$$

3.0 Method of Solution

The flow of a reactive Power-law fluid with variable thermal conductivity is governed by the continuity, momentum and energy equations. When the fluid is incompressible, neglecting the body forces and thermal conductivity is constant as contained in [7] the equations take the form

$$\rho \frac{DV}{Dt} = \nabla . S \tag{2.1}$$

S is a stress tensor for a Power-law fluid and defined as

$$S = -pI + \eta\left(trA_1^2\right)^n A_1 \tag{2.2}$$

When the reactant molecules rearrange to form the products, a chemical reaction occurs and thermal energy is produced. The volume rate of thermal energy, Q_c is in general a complicated function of pressure, temperature, composition and catalyst activity. It is assumed that the fluid behaves like a non-Newtonian fluid as far as the reactions are concerned. The energy equation takes the form

$$\rho \frac{de}{dt} = S.L - div\, q + \rho r + Q_c \tag{2.3}$$

where I is the unit tensor, η is the apparent viscosity, n is the flow behavior index, A_1 is the Rivlin Erickson tensor, e is the internal energy. We assume that the heat flux vector q is given by Fourier's law, $q = -K\nabla T$, where K is the thermal conductivity of the material, T is the temperature and Q_c is the chemical reaction term, $Q_c = QC_0K_0(T)$,

C_0 is the initial concentration of reactant species, Q is the heat energy and

$$K_0(T) = J\left(\frac{kT}{vh}\right)^m \exp\left(-\frac{E}{RT}\right),\ r = -\left(\frac{\partial q_r}{\partial y}\right),\ q_r = -\frac{4\sigma^*}{3k_1}\frac{\partial T^4}{\partial y}, T^4 \cong 4T_\infty^3 - 3T_\infty^4 \tag{2.4}$$

where J is the rate constant, k is the Boltzmann's constant, L is the velocity gradient, v is the vibration frequency, h is the Plank's number, r is the radiating energy, E is the activation energy, R is the universal gas constant and m is a numerical exponent.

Subject to the boundary conditions

$$u(-h,t) = 0, u(h,t) = U \tag{2.5}$$

$$T(0,t) = T_0, T(h,t) = T_0 \tag{2.6}$$

$$V = (u(y),0,0), T = T(y) \tag{2.7}$$

The continuity equation is satisfied and using the velocity profile (2.7) in the momentum and energy, we obtain

$$2^n\eta(2n+1)(u')^{2n}(u')^2 - \frac{\mu_{ef}v}{K} = \frac{\partial p}{\partial x} \tag{2.8}$$

$$\frac{d^2T}{dy^2}\left(\frac{16\sigma^* T_\infty^3}{3k_1} + k\right) + 2^n\eta(u')^{2n+2} + \frac{\mu_{ef}v^2}{K} + QC_0K_0(T) = \frac{\partial p}{\partial x} \tag{2.9}$$

Following [7, 8], we now introduce the dimensionless variables below

$$\bar{y} = \frac{y}{h},\ \bar{u} = \frac{u}{V}, C_1 = \frac{h}{2^n\eta}\left(\frac{h}{V}\right)^{2n+1}\frac{dp}{dx},\ Da = \frac{K}{Vh^2}, \varepsilon = \frac{RT_0}{E}, \theta = \frac{E(T-T_0)}{R{\theta_0}^2},\ R_d = \frac{4\sigma^* T_\infty^3}{k^* K}$$

$$\Lambda = \frac{\beta_* V^2}{\mu_* h^2},\ Br = \frac{\mu_* V^2}{k\varepsilon}\frac{2^n\eta h^2}{T_0}\left(\frac{V}{h}\right)^{2n+2}, \bar{\mu} = \frac{\mu}{\mu_*},\ \delta = \frac{k^m h^2 T_0^{m-2}}{V^m h^m RK\mu_* V^2}\frac{EQC_0A}{(RT_0^2 + T_0E)}e^{-E/RT_0} \tag{2.10}$$

Substituting (2.10) into (2.8) and (2.9) and dropping the prime, we obtain

$$\frac{d}{dy}(u')^{2n+1} - \frac{u}{Da} = C \tag{2.11}$$

$$\theta''\left(1 + \frac{4}{3}R_d\right) + \frac{u^2}{Da} + Br(u')^{2n+2} + \delta(1+\varepsilon\theta)^m e^{\frac{\theta}{(1+\varepsilon\theta)}} = 0 \tag{2.12}$$

The dimensionless boundary conditions:

$$u(-1)=0, u(1)=1,\ \theta(0)=0,\ \theta(1)=0 \tag{2.15}$$

We now solve equations (2.13) and (2.14) together with the boundary conditions (2.15) numerically using Galerkin-Weighted Residual Method as follows:

$$\text{Let}\, u=\sum_{i=0}^{2} A_i e^{y}, \theta=\sum_{i=0}^{2} B_i e^{\left(-i/4\right)y} \tag{2.16}$$

A maple 14 pseudo code was used to perform the iterative computation and results are presented in Figures 1and 2 as follows:

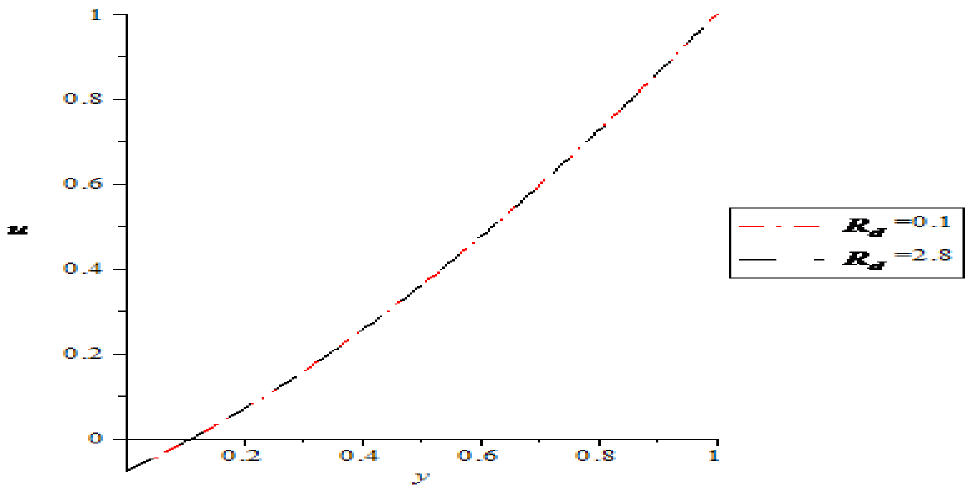

Figure 1: Graph of the velocity function u for various values of n when $C=1.0, Br=0.5, Da=2.0$

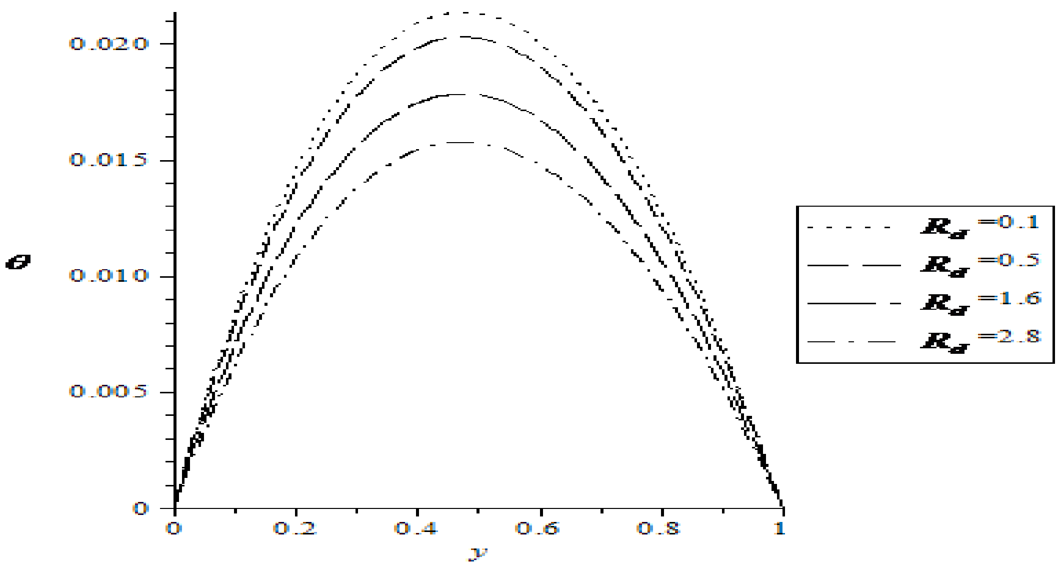

Figure 2: Graph of the temperature function θ against the similarity variable y of when $\Lambda=0.5,\ \varepsilon \geq 0, Br \geq 0, n \geq 0, Da=0.5.$

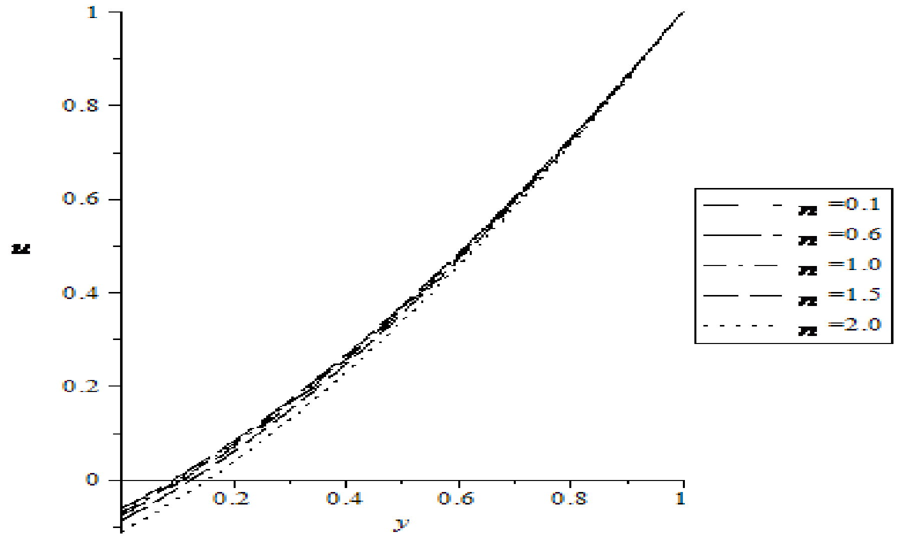

Figure 3: Graph of the velocity function u for various values of n when R_d
$C = 1.0, Br = 0.5, Da = 2.0$

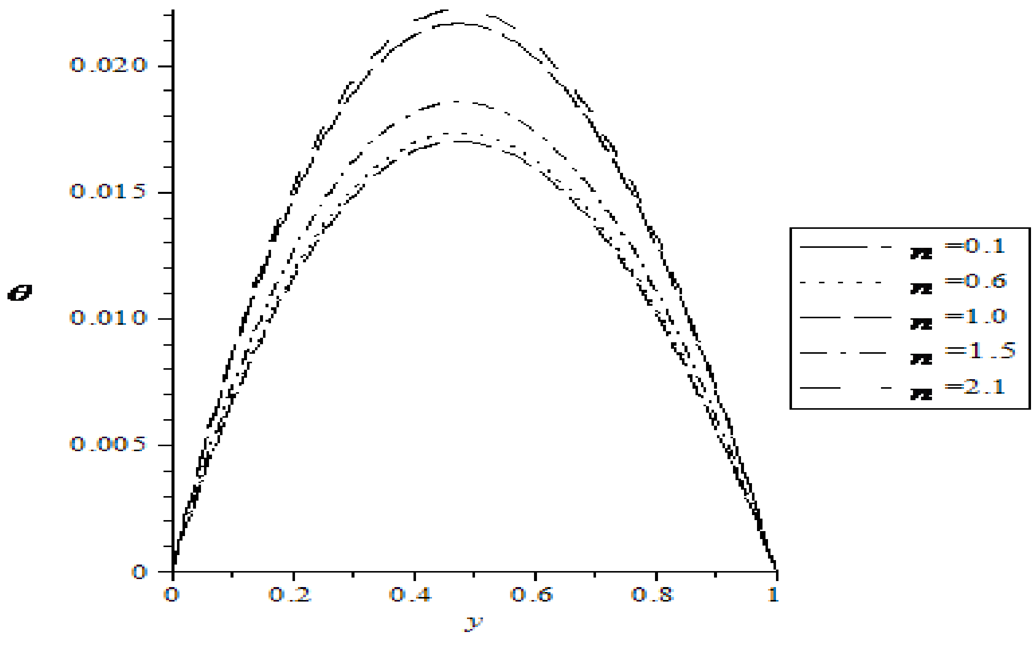

Figure 4: Graph of the temperature function θ against the similarity variable y of when $\Lambda = 0.5,\ \varepsilon \geq 0, Br \geq 0, n \geq 0, Da = 0.5.$

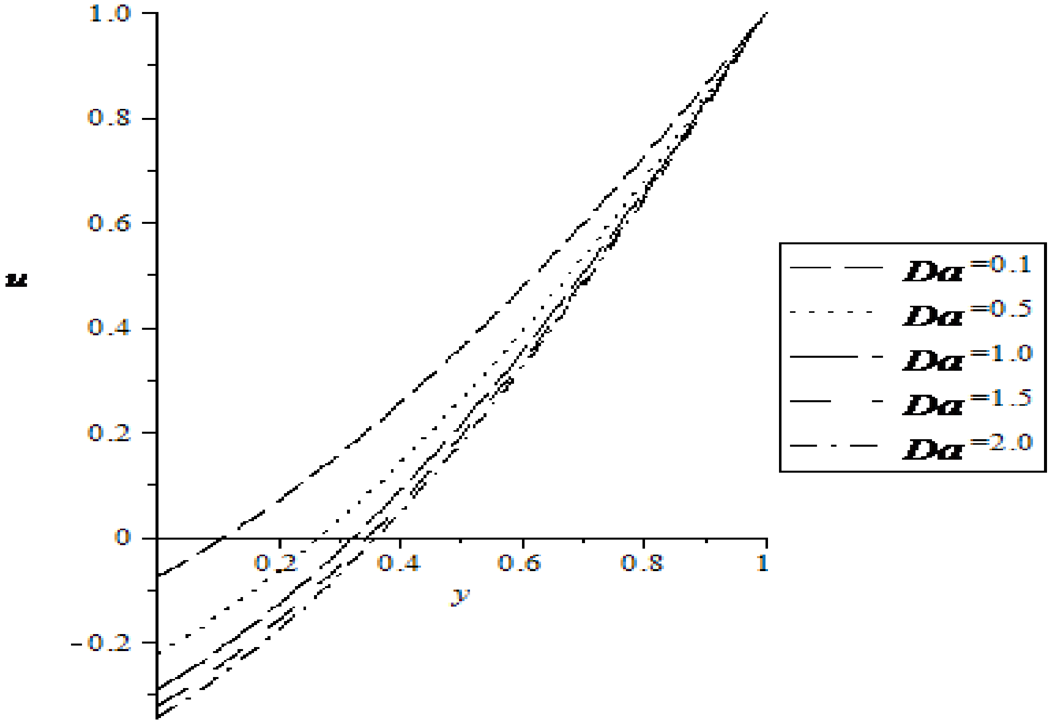

Figure 5: Graph of the velocity function u for various values of Da when $C = 2.0, Br = 1.5, \delta = 1.0$

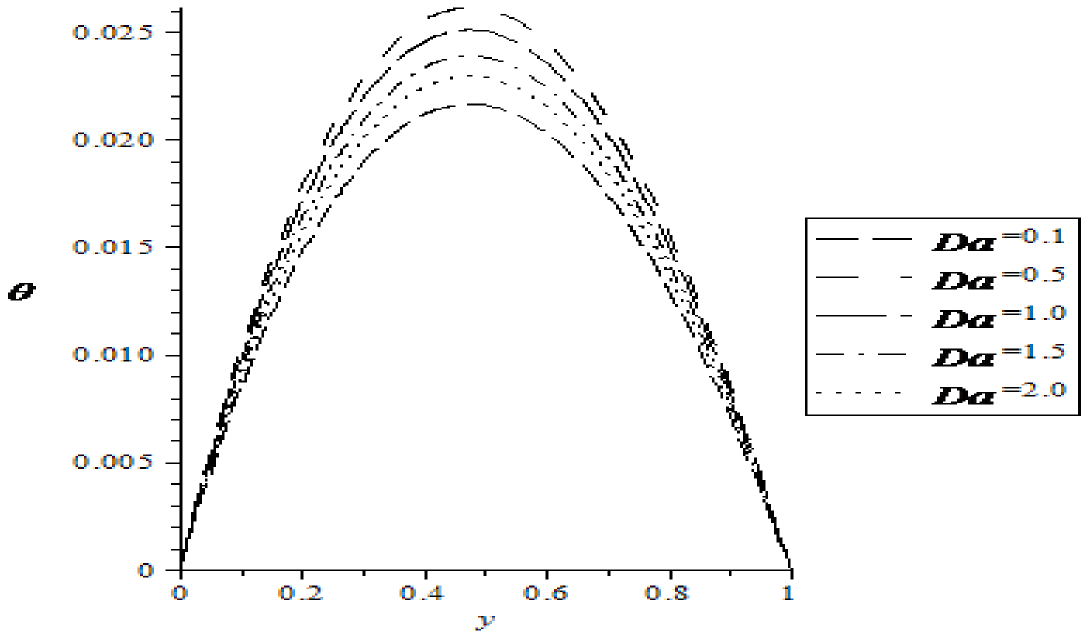

Figure 6: Graph of the temperature function θ against the similarity variable y of when $\Lambda = 1.5,\ \varepsilon \geq 0, Br \geq 0, n \geq 0, \delta = R_d = 1.0.$

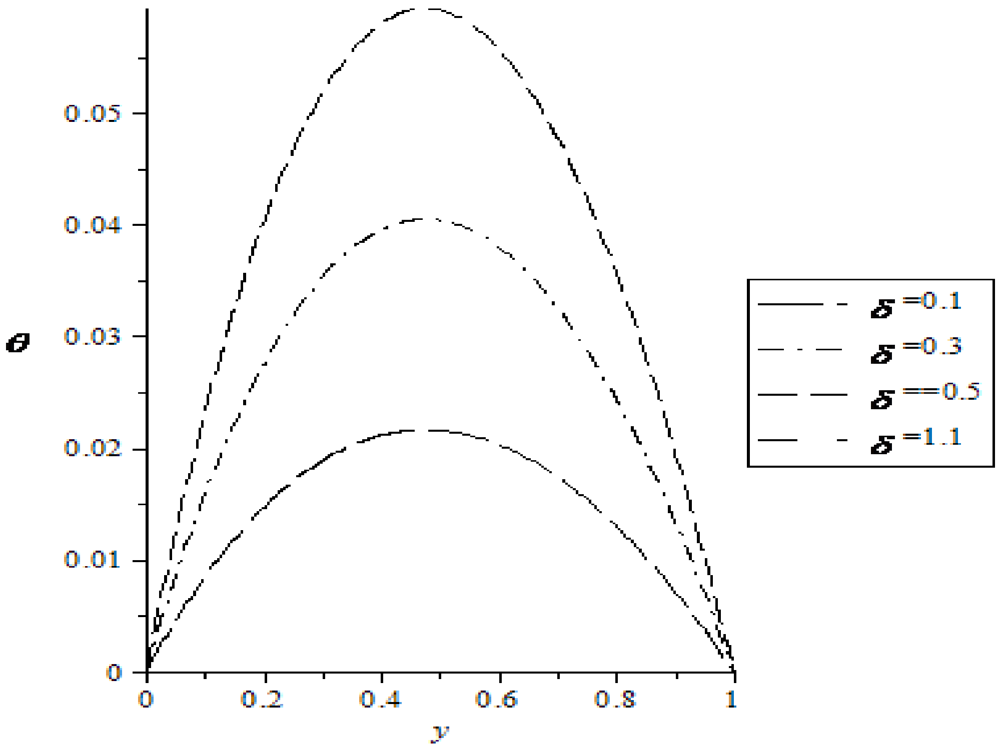

Figure 7: Graph of the temperature function θ against the similarity variable y of when $\Lambda = 1.5, \varepsilon \geq 0, Br \geq 0, n \geq 0, R_d = Da = 1.0.$

4.0 Discussion of Results/Conclusion

We consider a reactive power-law fluid with variable thermal radiation through a saturated porous medium. From Figures1,3 &5 the results show that the fluid velocity increases with increase in each of Power-law index n ,radiation parameter R_d and Darcy number Da . Figures 2, 4, 6 & 7 show that the temperature profile decreases with increase in each of Power-law index n , radiation parameter R_d , $Frank - Kamenetskii$ parameter δ and Darcy number Da .

Conclusion

It is observed that the fluid temperature decreases with increase in each of Power-law index n , radiation parameter R_d , $Frank - Kamenetskii$ $parameter\, \delta$ and Darcy number Da . We observed that there is transient increase in the fluid velocity with increase in each of Power-law index n ,radiation parameter R_d , δ $Frank - Kamenetskii$ $parameter\, \delta$, non-Newtonian parameter Λ , Brinkman number Br and Darcy number Da which decreases the porosity in the system of flow.

For engineering purpose, the flow model of our problem represents the oils well and as radiation parameter R_d is increasing there is quick recovery of oil from the oils well. Also, the results of this problem are of great interest in production processing, for the safety of life and proper handling of the materials during processing.

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