Int. J. Pure Appl. Sci. Technol., 24(2) (2014), pp. 29-38

International Journal of Pure and Applied Sciences and Technology

ISSN 2229 - 6107

Available online at www.ijopaasat.in

Research Paper

Second Law Analysis of Mass Transfer Effect on Unsteady MHD Flow Past an Accelerated Vertical Porous Plate

Okedoye Akindele Michael

Department of Mathematics and Computer Science, Federal University of Petroleum Resources, P. M. B. 1221. Effurun, Nigeria

Corresponding author, e-mail: (dele.mikeoke@gmail.com)

(Received: 11-8-14; Accepted: 24-9-14)

Abstract: *This paper reports the analytical calculation of entropy generation due to unsteady heat and mass transfer on MHD flow past an infinite vertical oscillating plate, taking account of the presence of free convection and mass transfer. The fluid and the plates are in a state of solid body rotation with constant angular velocity about the z-axis normal to the plates. The governing equations are solved analytically and using perturbation technique. The influences of flow parameters such as porosity parameter (α), Grashof numbers (Grt, Grc), Hartmann's number (M), heat generation/absorption (β) and reaction parameter (γ) on total entropy generation were investigated, reported and discussed.*

Keywords: Free convection, Magneto hydrodynamic flows, Porous medium, Oscillating plate, Entropy generation.

---

1. Introduction

The study of convective flow with mass transfer along a vertical porous plate isreceiving considerable attention of many researchers because of its varied applications in the field of cosmical and geophysical sciences. Permeable porous plates are used in the filtration processes and also for a heated body to keep its temperature constant and to make the heat insulation of the surface more effective. The study of stellar structure on the solar surface is connected with mass transfer phenomena. Its origin is attributed to difference in temperature caused by the non-homogeneous production of heat, which in many cases can rest not only in the formation of convective currents but also in violent explosions. Mass transfer certainly occurs within the mantle and cores of planets of the size of or larger than the earth. It is therefore interesting to investigate this phenomenon and to study in particular, the case of mass transfer on the free convection flow [1].

Flow past a vertical plate oscillating in its own plane has many industrial applications. The first exact solution of Navier-Stokes equation was given by Stokes [2] which is concerned with flow of viscous incompressible fluid past an horizontal plate oscillating in its own plane. Natural convection effects on Stokes problem was first studied by Soundalgekar [3].

For a given system, a set of thermodynamic parameters, which optimize the operating conditions, may be obtained. Nag and Kumar [4] studied second Law optimization for convective heat transfer through a duct with constant heat flux. In their study, they plotted the variation of entropy generation versus the temperature difference of the bulk flow and the surface using a duty parameter. Shuja and Yilbas [5] analyzed the entropy generation in an impinging jet and Shuja et al. ([6], [7], [8]) consider swirling jet impingement on an adiabatic wall for various flow conditions. The dissipation of energy takes the form of a sum of products of conjugate forces and fluxes associated to the problem under consideration; this was presented by the text of the De Groot S. R [9]. The fluxes are expressed as linear functions of all forces, as constitutive equations, subjected to the reciprocal relations of Onsager. These lead to coupled field equations for the temperature and species concentrations in a given fluid mixture. Interferences between heat and mass transport, at the level of constitutive equations, and the linear theory of non-equilibrium thermodynamics had been formulated as a constitutive theory capable of fully expressing the dependence of all fluxes as a function of all thermodynamic forces. Entropy generation in Magneto Hydro Dynamic (MHD) flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction was studied and reported by Okedoye et al [10] many researchers have worked on entropy generation and Okedoye et al [11] has a good review of some of this works.

The present paper reports an analytical determination of the entropy generation of unsteady two-dimensional heat and mass transfer effect on MHD free convection flow past an oscillating plate in the presence of heat generation/absorption and chemical reactionwhen the plate accelerates in its own plane.

2. Formulation of the Problem

Consider the unsteady flow of an incompressible viscous fluid past an acceleratingvertical porous plate. Let the *x*-axis be directed upward along the plate and the*y*-axis normal to the plate. Let u and v be the velocity components along the*x*- and *y*- axes respectively. Let us assume that the plate is accelerating with avelocity $u = v_0$in its own plane at time $t=0$. Then the unsteady boundary layer equations in the Boussinesq's approximation, together with Brinkman's empiricalmodification of Darcy's law, are (as given by Okedoye [11])

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - \frac{v}{A}u + \frac{g\beta_\tau}{\rho}(T - T_\infty) + \frac{g\beta_c}{\rho}(C - C_\infty) \tag{2.2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty) \tag{2.3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - R(C - C_\infty) \tag{2.4}$$

where k is the thermal diffusivity, υ is the kinematics viscosity, $A$is the permeability coefficient, β_τ is the volumetric expansion coefficient for heat transfer, β_c is the volumetric expansion coefficient for mass transfer, ρ is the density, g is theacceleration due to gravity, T is the temperature, T_∞ is the temperature of the fluid far away from the plate, C is the concentration, C_∞ is the concentration faraway from the plate and D is the molecular diffusivity.

On disregarding the Joulean heat dissipation, the boundary conditions are given by

$$\left.\begin{array}{r} u=0,\;\; T=T_\infty,\, C=C_\infty \;\; for\; all\;\; y,\, t\le 0 \\ u=v_0,\, v=-v_0, T=T_w+\varepsilon e^{i\omega t},\, C=C_w+\varepsilon e^{i\omega t},\; y=0,\, t>0 \\ u=0,\, T=T_\infty,\, C=C_\infty \;\; as\;\; y\to\infty,\, t>0 \end{array}\right\} \tag{2.5}$$

We consider the fluid lying in the upper half space/plane. The x − axis is taken along the flow direction and y − axis perpendicular to it: such that there is simultaneous suction/blowing at the boundary y = 0. In fact, it follows from the continuity equation (2.1) that

$$\frac{\partial v}{\partial y}=0$$

which implies $v=-v_w = const$ so that the velocity field takes the form

$$V=(u(y,t),v_w) \tag{2.1.1}$$

Let us introduce the non-dimensional variables

$$u'=\frac{u}{u_w},\; t'=\frac{tv_w^2}{\upsilon},\; y'=\frac{yv_w}{\upsilon},\; \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{2.1.2}$$

where all the physical variables have their usual meanings.

With the help of (2.1.1) and (2.1.2), on dropping primes (′) the governing equations (2.1) – (2.4) with the boundary conditions (2.5) reduce to

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+Gr\tau\theta+Grc\phi-\left(M+\frac{1}{\alpha}\right)u \tag{2.6}$$

$$\frac{\Pr}{4}\frac{\partial\theta}{\partial t}-\Pr\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial y^2}+\Pr\beta\theta \tag{2.7}$$

$$\frac{Sc}{4}\frac{\partial\phi}{\partial t}-Sc\frac{\partial\phi}{\partial y}=\frac{\partial^2\phi}{\partial y^2}-\gamma Sc\phi \tag{2.8}$$

$$\left.\begin{array}{l} u=0,\;\; \theta=0,\;\; \phi=0,\;\; for\, all\;\; y,\;\; t\le 0 \\ u=1,\, \theta=1+\varepsilon e^{i\omega t},\;\; \phi=1+\varepsilon e^{i\omega t},\; y=0,\, t>0 \\ u\to 0,\;\; \theta\to 0,\;\; \phi\to 0,\; as\, y\to\infty,\;\; t>0 \end{array}\right\} \tag{2.9}$$

Where the parameters are as defined below:

$$\left.\begin{array}{l} Gr\tau=\dfrac{g\beta_\tau(T_w-T_\infty)v}{v_0^3},\;\; Grc=\dfrac{g\beta_c(C_w-C_\infty)v}{v_0^3},\;\; M=\dfrac{\sigma B_0^2 v}{\rho v_0^2} \\ \alpha=\dfrac{v_0^2 A}{v^2},\;\Pr=\dfrac{\mu c_p}{k},\;\; \beta=\dfrac{Q\mu v}{kv_0^2\rho},\;\; Sc=\dfrac{v}{D},\;\; \gamma=\dfrac{Rv}{v_0^2} \end{array}\right\}$$

Where *Pr, Sc, Grc, Grt,* α, β, γ and M are Prandtl number,Schmidt number,Grashof number for mass transfer,Grashof number for heat transfer,Porosity parameter, heat generation/absorption, Chemical reaction parameter and Hartmann's number.

3. Method of Solution

To solve the problem posed in equations (2.1) – (2.4), we seek a perturbation series expansion in the limit of ε for our dependent variables. This is justified since ε is small; we have

$$\left.\begin{aligned} u(y,t) &= u_0(y) + \varepsilon e^{i\omega t} u_1(y) + o(\varepsilon^2) + \ldots \\ \theta(y,t) &= \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y) + o(\varepsilon^2) + \ldots \\ \mathrm{H}(y,t) &= \mathrm{H}_0(y) + \varepsilon e^{i\omega t} \mathrm{H}_1(y) + o(\varepsilon^2) + \ldots \end{aligned}\right\} \tag{3.1}$$

Substituting equations (3.1) the equations (2.6) - (2.9) and equating the harmonic and non – harmonic terms and neglecting the coefficient of ε^2 to obtain a set of equations for velocity, concentration and the temperature fields. The solutions of which are

$$\begin{aligned} &\phi_0(y) = e^{-ny},\ \ \theta_0(y) = e^{-my},\ \ u_0(y) = a_4 e^{-ry} + a_5 e^{-my} + a_6 e^{-ny} \\ &\phi_1(y) = e^{-i\omega t} e^{-n_1 y},\ \ \theta_1(y) = e^{-i\omega t} e^{-m_1 y},\ \ u_1(y) = a_8 e^{-r_1 y} + a_9 e^{-m_1 y} + a_{10} e^{-n_1 y} \end{aligned} \tag{3.2}$$

where

$$m = \frac{1}{2}\left(\Pr + \sqrt{\mathbf{Pr}^2 - 4\mathbf{P}r\beta}\right),\ \ m_1 = \frac{1}{2}\left(\Pr + \sqrt{\Pr^2 - 4\Pr\left(\beta - \frac{i\omega}{4}\right)}\right),$$

$$n = \frac{1}{2}\left(Sc + \sqrt{Sc^2 + 4Sc\gamma}\right),\ \ n_1 = \frac{1}{2}\left(Sc + \sqrt{Sc^2 + 4Sc\left(\gamma + \frac{i\omega}{4}\right)}\right),$$

$$r = \frac{1}{2}\left(1 + \sqrt{1 + 4\left(M + \frac{1}{\alpha}\right)}\right),\ r = \frac{1}{2}\left(1 + \sqrt{1 + 4\left(M + \frac{1}{\alpha} + \frac{i\omega}{4}\right)}\right),$$

With

$$a_4 = 1 - a_5 - a_6,\ a_8 = -a_9 - a_{10},\ a_5 = \frac{-Grt}{m^2 - m - \left(M + \frac{1}{\alpha}\right)},\ a_6 = \frac{-Grc}{n^2 - n - \left(M + \frac{1}{\alpha}\right)},$$

$$a_9 = \frac{-Grt \cdot e^{i\omega t}}{m^2 - m - \left(M + \frac{1}{\alpha} + \frac{i\omega}{4}\right)},\ \ a_{10} = \frac{-Grc \cdot e^{i\omega t}}{n^2 - n - \left(M + \frac{1}{\alpha} + \frac{i\omega}{4}\right)}$$

Where the functions $u_0(y)$, $\theta_0(y)$ and $\phi_0(y)$ are the mean velocity, the mean temperature and the mean concentration fields, respectively; and $u_1(y)$, $\theta_1(y)$ and $\phi_1(y)$ are, respectively, the velocity oscillatory part, the temperature oscillatory part and the concentration oscillatory partfields.

Now substituting equations (3.2) into equation (3.1), we obtain the required expressions for velocity, temperature and magnetic induction;

$$u(y,t) = a_4 e^{-ry} + a_5 e^{-my} + a_6 e^{-ny} + \varepsilon e^{i\omega t}\left(a_8 e^{-r_1 y} + a_9 e^{-m_1 y} + a_{10} e^{-n_1 y}\right) \quad (3.3)$$

$$\theta(y,t) = e^{-my} + \varepsilon e^{2i\omega t} e^{-m_1 y} \quad (3.4)$$

$$\phi(y,t) = e^{-ny} + \varepsilon e^{2i\omega t} e^{-n_1 y} \quad (3.5)$$

4. Entropy Generation Rate

For an incompressible Newtonian fluid, the local entropy generation rate is given by Okedoye [10] as:

$$\Gamma = \frac{\mu}{T}\left(\frac{\partial u_i}{\partial x_j}\right)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{1}{T}\sum_{\alpha} J\alpha_i\left(\frac{\partial u_\alpha}{\partial x_i}\right) - \frac{q}{T^2}\left(\frac{\partial T}{\partial x_i}\right)$$
$$- \frac{1}{T}\sum_{\alpha} S_\alpha J\alpha_i\left(\frac{\partial u_\alpha}{\partial x_i}\right) - \frac{1}{T}\sum_{\alpha} K_\alpha \mu_\alpha$$

On the right hand side of the above equation, the first term is due to fluid friction, the second is due to mass diffusion and the third term is due to heat conduction. The fourth term is due to heat transfer induced by mass diffusion and the fifth is due to chemical reactions.

Okedoye et al [10] defined the two –dimensionless entropy generation rate as

$$\Gamma_n = \left(\frac{\partial \theta}{\partial y}\right)^2 + \lambda_1\left(\frac{\partial u}{\partial y}\right)^2 + \lambda_2\left(\frac{\partial c}{\partial y}\right)^2 + \lambda_3\left(\frac{\partial \theta}{\partial y}\right)\left(\frac{\partial c}{\partial y}\right) \quad (4.1)$$

Where

$$\Gamma_{n,h} = \left(\frac{\partial \theta}{\partial y}\right)^2, \Gamma_{n,f} = \lambda_1\left(\frac{\partial u}{\partial y}\right)^2, \Gamma_{n,d}^{c} = \lambda_2\left(\frac{\partial c}{\partial y}\right)^2, \text{ and } \Gamma_{n,d}^{c,T} = \lambda_3\left(\frac{\partial \theta}{\partial y}\right)\left(\frac{\partial c}{\partial y}\right),$$

where $\Gamma_{n,h}$ and $\Gamma_{n,f}$ are thermal and viscous irreversibility respectively, while $\Gamma_{n,d}^{c} + \Gamma_{n,d}^{c,T}$ is the diffusive irreversibility

Dimensionless terms denoted $\lambda_i\ (1 \le i \le 3)$, and called irreversibilities distribution ratios, are given by:

$$\lambda_1 = \frac{\mu T_0}{k}\left(\frac{a}{L(\Delta T)}\right)^2, \quad \lambda_2 = \frac{RDT_0}{kc_0}\left(\frac{\Delta c}{\Delta T}\right)^2, \quad \lambda_3 = \frac{RD}{k}\left(\frac{\Delta c}{\Delta T}\right) \quad (4.2)$$

where C_0 and T_0 are respectively the reference concentration and temperature, which are in our case, the bulk concentration and the bulk temperature.

The local entropy generation rate is a function of concentration temperature and velocity gradients in the *y* directions in the entire calculation domain.

Using the above equation, on substituting equations (3.3) to (3.5) for irreversibilities in (4.1), we have

$$\left.\begin{aligned}\Gamma &= \left(me^{-my} + \varepsilon m_1 e^{2i\omega t - m_1 y}\right)^2 \\ &\quad + \lambda_1 \left(a_4 r e^{-ry} + a_5 m e^{-my} + a_6 n e^{-ny} + \varepsilon e^{2i\omega t}\left(a_8 r_1 e^{-r_1 y} + a_9 m_1 e^{-m_1 y} + a_{10} n_1 e^{-n_1 y}\right)\right)^2 \\ &\quad + \lambda_2 \left(ne^{-ny} + \varepsilon n_1 e^{2i\omega t - n_1 y}\right)^2 + \lambda_3 \left(me^{-my} + \varepsilon m_1 e^{2i\omega t - m_1 y}\right)\cdot\left(ne^{-ny} + \varepsilon n_1 e^{2i\omega t - n_1 y}\right)\end{aligned}\right\}$$

5. Discussion

In order to point out the effects of various parameters on flowcharacteristic, the following discussion is set out. The values of the Prandtl number is chosen Pr = 0.71 (plasma). The values of the Schmidtnumber is chosen to represent the presence of species by water vapour (0.60). All other parameters are primarily chosen as follows: $Gr\tau = 10,\ Grc = 5,\ \alpha = 5,\ B = -2,\ M = 0.5,\ K = 1.5,$ $\omega t = 2\pi,\ t = 0.25,$ unless otherwise stated. We carried out the analysis on both a case of heated plate and the case of cooling of the plate

Figure 1 depicts the effect of thermal Grashof number on the entropy generation. It is observed that increase in thermal Grashof number results in decrease in entropy generation for cooling of the plate, while an increase in thermal buoyancy brings about an increase in the entropy generation. For small thermal Grashof number, there is practically little or no convection and the entropy generation due to fluid friction is zero, consequently the total entropy generation is reduced to the entropy generation due to heat transfer. At higher Grashof number heat transfer due to convection begins to play a significant role increasing the flow velocity and in turn the entropy generation due to the viscous effects. Also the isotherms are deformed increasing the temperature gradient and consequently the entropy generation due to heat transfer. For either heating or cooling of the plate, entropy is highest at the surface and decay away along the channel of flow. The lowest entropy occurs when the thermal buoyancy is zero, but comparing the values of entropy generated at position y=0.2, entropy in the heating of plate is far higher than that of cooling of plate.

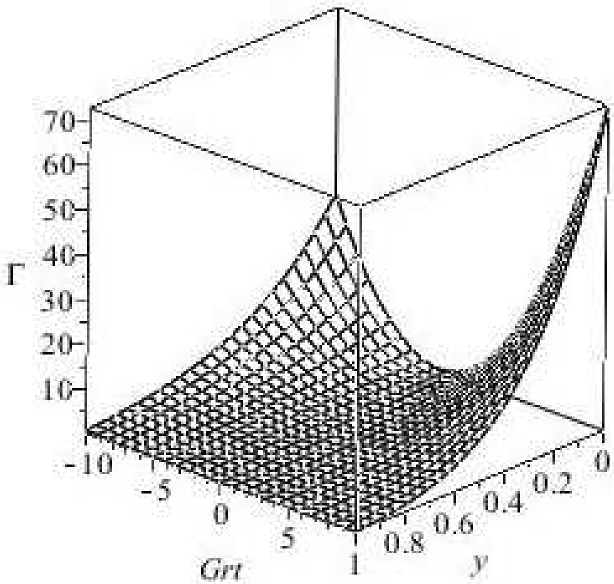
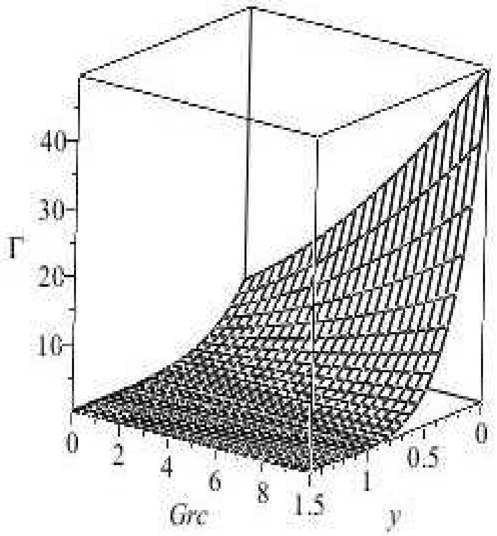


Figure 1: Effect of Thermal Buoyancy on Entropy distribution



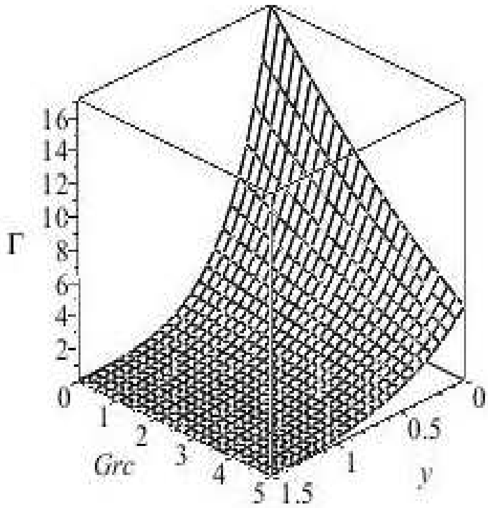
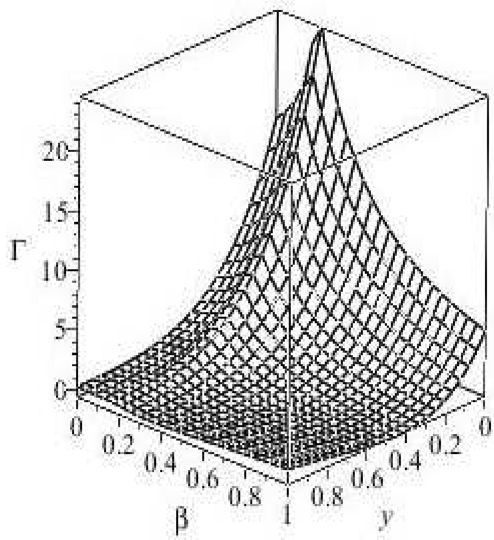


Figure 2a **Figure 2b**

Figure 2a and 2b: Entropy distribution with mass buoyancy

Figures 2(a) and 2(b) displayed the effect of mass Grashof number on the entropy generation during the flow process. It isobserved that in the case of cooling of surface, an increase in *Grc*result in decrease in total entropy generated (Figure 2(b)) while for heating of the plate entropy increases with an increase in mass Grashof number. It could be seen from the figures that cooling of the plate results in lower entropy generation (Figure 2(b)) while heating ofthe plate increase the entropy (Figure 2(a)).



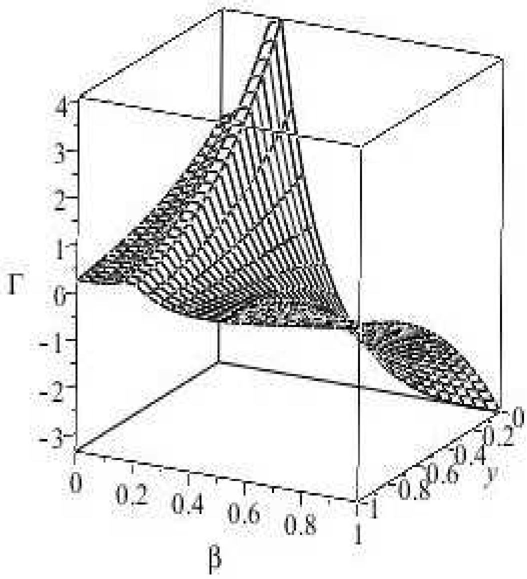
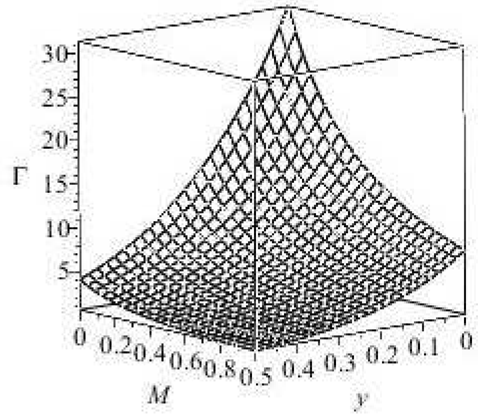


Figure 3a **Figure 3b**

Figure 3 shows the effect of heat generation or absorption on the entropy generation. It could be seen that increase in heat absorption results in increase in entropy generation, while increase in heat generation reduce the total entropy generated for both cooling and heating of the plate. Maximum entropy is noted to occur at in both cases.



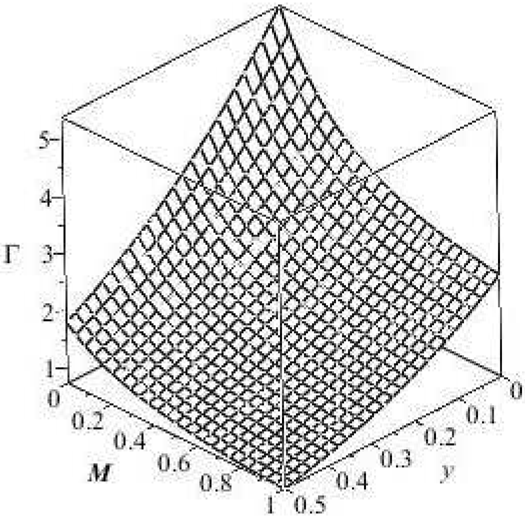
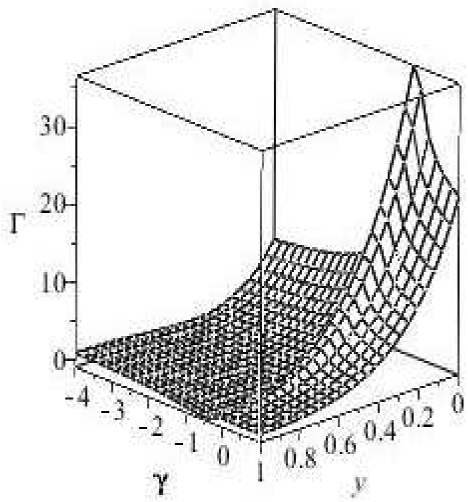


Figure 4a **Figure 4b**

In figure 4, we show the effect of magnetic parameter on the entropy generation. It could be seen that entropy reduces with an increase in Hartmann's number. This is because the entropy generation depends on the rate of disorderliness which in turn depends on the magnitude of the velocity. The induced opposing force, the Lorentz force brings about reduction in the velocity and hence the entropy. It could also be seen that the entropy is more pronounced in the case of cooling of the plate $(Grt < 0)$, than in the case of heating of the plate $(Grt > 0)$



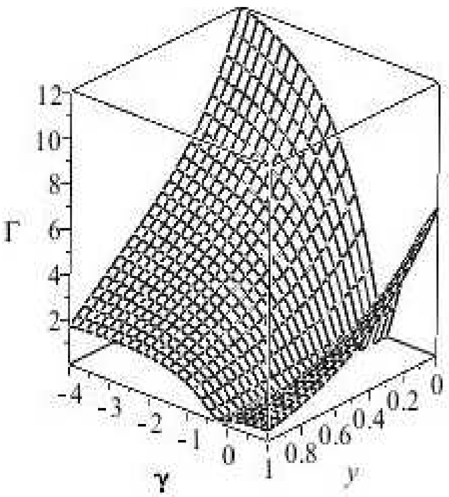
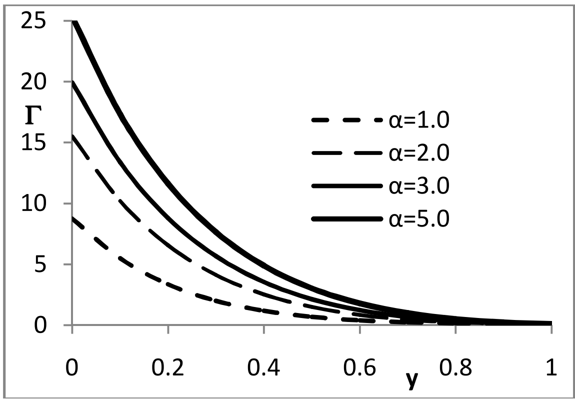


Figure 5a **Figure 5b**

Figure 5: Entropy generation for various values of γ

In figure 5, we show the effect of reaction parameter γ on the entropy generation. We also noted that correspond to generative chemical reaction, while correspond to destructive chemical reaction. It could be seen that increase in destructive chemical reaction results in lowers the entropy generation for the case of cooling of the plate (Figure 5(a)), while increase in destructive chemical reaction increases the total entropy generated Figure 5(b).



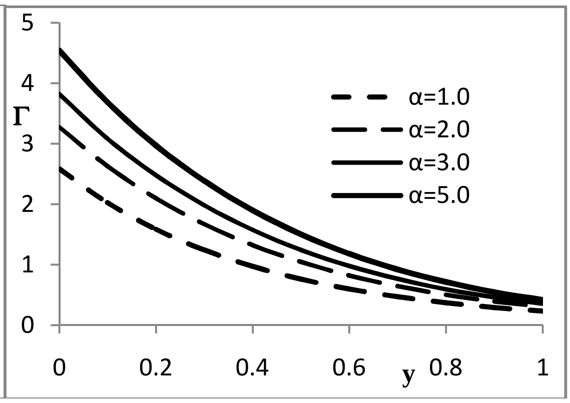


Figure 6a **Figure 6b**

Figures 6 a) and (b) depict the effect of permeability parameter on the entropy generation. We observed that entropy generation increases with an increase in permeability factor for both cooling and heating of the plate. This is due to the fact that increase in the value of α has thetendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow.

6. Conclusions

In this paper, entropy generation of unsteady two-dimensional heat and mass transfer effect on MHD free convection flow past an oscillating plate in the presence of heat generation/absorption and chemical reactionis presented. Results are presented graphically to illustrate the variation in entropy as a result of variation in the flow control parameters. In this study, the following conclusions are set out: In case of cooling of the plate (Grt> 0), the entropy decreases with an increase in magnetic parameter and heat generation, Grc and destructive chemical reactivity. On the other hand, for heating of the plate, it increases with an increase in the value of heat absorption coefficient, mass Grashof number and permeability parameter. Entropy is higher in heating of the plate with M, β, and Grc, while it is higher in cooling of the plate with α and γ

Acknowledgements

The author wish to thank the Vice Chancellor, Federal University of Petroleum Resources, Effurun and his management team for providing and enabling environment, in the area of internet facility and electricity throughout the period of this research. Also, the Department of Mathematics and Computer Science, Federal University of Petroleum Resources, Effurun is appreciated for permission to use its computer laboratory.

References

[1] S.S. Das, S.K. Shaoo and G.C. Dash, Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction, *Bull. Malays. Math. Sci. Soc.*, 29(1) (2006), 33-42.

[2] S.R. De Groot, Thermodynamics of Irreversible Processes, North-Holland, Amesterdam, 1966.

[3] A.M. Okedoye, O.T. Lamidi and R.O. Ayeni, Entropy generation in MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction, *The Journal of Mathematical Association of Nigeria*, 34(2AMathematical Series) (2007), 52-57.

[4] G.G. Stokes, On the effect of the internal friction of fluid on the motion of pendulum, *Transactions Cambridge Philosophical Society*, IX(1851), 8-106.

[5] V.M. Soundalgekar, Free convection effects on the flow past a vertical oscillating plate, *Astrophysics and Space Science*, 64(1979), 165-172.
[6] S.Z. Shuja, B.S. Yilbas and M.O. Budair, Local entropy generation in an impinging jet: Minimum entropy concept evaluating various turbulence models, *Computer Methods in Applied Mechanics and Engineering*, 190(2001), 3623-3644.
[7] S.Z. Shuja, B.S. Yilbas and M.O. Budair, Investigation into a confined laminar swirling jet and entropy production, *Int. J. Numerical Methods for Heat and Fluid Flow*, 12(November) (2002), 870-887.
[8] S.Z. Shuja and B.S. Yilbas, A laminar swirling jet impingement on to an adiabatic wall effect of inlet velocity profiles, *Int. J. Numerical Methods for Heat and Fluid Flow*, 11(2001), 237-254.
[9] S.Z. Shuja, B.S. Yilbas and M. Rashid, Confined swirling jet impingement onto an adiabaticwall, *Int. J. Heat Mass Transfer*, 46(2003), 2947-2955.
[10] P.K. Nag and N. Kumar, Second law optimization of convective heat transfer through a duct with constant heat flux, *Int. J. Energy Res.*, 13(1989), 537-543.