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Research Paper

Optimization of the Utility of a Structural Model of the Demand for Multi-Destination Non-Work Travel Using Maximum Entropy Method

Kakali Karmakar (Sur) [1,*] **and Sanat Kumar Majumder** [2]

[1] Department of Mathematics, B.P.Poddar Institute of Management and Technology, Poddar Vihar,137,V.I.P. Road,Kolkata-52, West Bengal, India.

[2] Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah-711103, West Bengal, India.

*Corresponding author email: (kakali_ks@rediffmail.com)

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Abstract: *In this paper we maximize the utility of a structural model of the demand for multi-destination non-work travel using maximum entropy method introduced by Shanon. A utility value is assigned when only partial information is available about the decision maker's preferences. The maximum entropy utility solution embeds a large family of utility functions that includes the most commonly used functional form. The model presented here incorporate travel frequency, destination choice and mode choice for both single and multi-destination travel into a unified utility-maximizing framework.*

Keywords: Optimization, Utility, Entropy, Non-work travelers, Destination, Mode of travel.

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1. Introduction

In recent years utility maximization has emerged as a fundamental behavior principle of urban passenger travel demand modeling. According to this principle, as individual's preferences for the travel options he faces, can be described by a utility function, and each individual chooses the option that chooses the option that maximizes his utility. In this paper we use the maximum entropy method to maximize his utility of travel options [4,5].

The planning of this paper is as follows:
2. [A].Development of the model. [4]
2. [B]. Maximization of utility using entropy maximization method [5]

2. Modeling and Optimization

2. [A]. Development of the model

The development of satisfactory utility maximiziing the model [4] of the demand for multi-destination non-work travel has proved to be considerably more difficult than the development of work trip mode choice model. This is due mainly to the variety and complexity of the travel options available to non-work travelers. These options typically include the frequency, destination and mode of travel, among other factors. The models of Domencich and Mc Fadden (1975), Adler and Ben-Akiva (1976) and Charles River Associates(1976) permit non-work(complete home to home round trips to non-work destinations) per household per day and do not permit multi-destination tours all are subsumed in this model.[3].

Let A be the set of non-work trips that are available to the members of a household. The trip ijm is an element of A if iand j is non-work locations (possibly including home) available to the household, and m is a mode that is available to the household for travel between iand j.It is not required that iand j be distinct. Let t be the time of day and let Δt be a time interval sufficiently short that the members of the household can begin at most one trip during the time period t to t+Δt.

Let $A_t \subset A$ be the set of trips that household members can start during t to t+Δt. The travel options available to the household during t to t+Δt are

<u>Option 1:</u> Begin a person trip from origin i to destination j by mode m (ijm$\in A_t$) as part of a tour from home to one or more non-work destination to home.

<u>Option 2:</u> Do not begin non-work travel that is part of a home non-work home tour.
A utility values is associated with each of these options. The household is assumed to choose the option with the highest utility.

The utilities are given the following functional representations:

<u>Option 1:</u> $U_{ijm}(x, s, z, N_0, t, \Delta t) + \varepsilon_{ijm}$

<u>Option 2:</u> $U_0(s, N_0, t, \Delta t) + \varepsilon_0$

Where: U=deterministic component of utility.
x =a vector of transportation level-of-service variables relevant to the choice ijm.
s= a vector of household characteristics.
z = a vector of destination characteristics, other than transportation level-of-service that are relevant to the choice ijm.
N_0=number of person trips.
ε = random component of utility.
N_0 is included in the utility function in order to capture the effects of past travel decisions and future travel plans on current travel decisions.

To deduce the structure of a demand model from the utility functions and the principle of utility-maximization it is necessary to specify the probablity distribution of the random utility component ε.

F (ε) =exp [-exp (-ε)]. (1)

Where F is the cumulative probablity distribution function of the random variable ε.

The probablity that a household member chooses to begin the trip ijm$\in A_t$ during t to t+Δt is:

$$P(ijm/N_0,t,\Delta t,A_t)=\frac{\exp(U_{ijm})}{[\exp(U_0)+\sum_{pqr\in A_t}\exp(U_{pqr})]}, \quad (2)$$

as Δt approaches zero then the probablity of travel during t to t+Δt also approaches zero and for small Δt, the probablity of the trip ijm$\in A_t$ is proportional to Δt

$$p(ijm/N_0,t,\Delta t,A_t)=P(ijm/N_0,t,A_t)\Delta t. \quad (3)$$

This implies that for small Δt the deterministic components of utility can be represented as:

$$U_{ijm}(x,s,z,N_{0,}t,\Delta t)=V_{ijm}(x,s,z,N_0,t)+\log\Delta t$$
$$U_0(s,N_0,t,\Delta t)=V_0(s,N_0,t) \quad (4)$$

Thus for small Δt

$P(ijm/N_0,t,A_t)\Delta t$ = [exp(V_{ijm}-V_0)] Δt. (5)

or, $P(ijm/N_0,t,A_t)\Delta t$ =[exp(V_{ijm})] Δt (absorbing V_0 intoV_{ijm})

Now the marginal probablity of choosing ijm:

$$P(ijm/N_0,t)\Delta t=P(ijm/N_0,t,A_t)P_r(ijm\in A_t)\Delta t$$
$$P(ijm/N_0,t)\Delta t=(\exp V_{ijm})P_r(ijm\in A_t)\Delta t \quad (6)$$

$P_r(ijm/N_{0,t})\Delta t$

should be a decreasing function of N_0 if i is home and also it is not known a prior.

$$P_r(ijm\in A_t/i=\text{home})=B(s,t)[1-k(s,t)N_0]. \quad (7)$$

B and k are functions of the household characteristics and time of day t.

Now let i be a non- home location then,

$$P_r(ijm\in A_t/i\neq\text{home})=L_{im}(x,s,z)N_{im,0}, \quad (8)$$

P(ijm/t) $P(ijm/t)\Delta t=\{\Delta t N_{im}L_{im}\ (\exp V_{ijm}^{(1)})\}$; i$\neq$home (9)

$\Delta t(\exp V_{ijm}^{(0)}+1+N[\exp(V_{ijm}^{(1)}-V_{ijm}^{(0)})-1]$; i= home

Since at most one non-work trip can begin during t to t+Δt,P(ijm/t) Δt is equal to the average numbers of trips from i to j by mode m that start during t to t+Δt.The average numbers of trips per day from i to j by mode m,N_{ijm} therefore can be obtained by integration

$$N_{ijm}=\int P(ijm/t)dt. \quad (10)$$

Where the integral extends over a day. Note that N_{ijm} and N are related by

$$N = \sum_{\substack{ijm \\ j \neq h}} N_{ijm} . \tag{11}$$

Where the subscript h signifies home. For r equal to 0 or 1,define $W_{ijm}^{(r)}$ by

$$W_{ijm}^{(r)} = [\log \int \exp(V_{ijm}^{(r)}) dt] . \tag{12}$$

$W_{ijm}^{(r)}$ is the time integrated utility of travel from i to j by mode m,given that r other trips to non-work destinations are made during the day.

Using equation (10), (11), (12) and integration of (9) over a day yields

$$N_{ijm} = N_{im} L_{im} (\exp W_{ijm}^{(1)}); i \neq \text{hom}\, e \tag{13}$$

$$N_{hjm} = (\exp W_{hjm}^{(0)})\{1 + N[\exp(W_{hjm}^{(1)} - W_{hjm}^{(0)}) - 1]\} . \tag{14}$$

If a household has j non-work destinations and M modes available to it, then (11), (13), (14) define a system of MJ(J+1)+1 equation in the MJ(J+3)+1 unknown quantities N,N_{ijm},N_{im},N_{hjm} and L_{im} where i,j and m range over the available destinations and modes. To obtain a unique solution for the unknown quantities, it is necessary to add 2Mj equation to the system represented by (11).(13),(14) we get

$$N_{im} = \sum_{j} N_{ijm} ; i \neq \text{hom}\, e , \tag{15}$$

$$\sum_{i} N_{jim} = \sum_{i} N_{ijm} ; j \neq \text{hom}\, e . \tag{16}$$

Equation(11)&(13),through (16) constitute a system that is solvable for N,N_{ijm},N_{im},N_{hjm} and L_{im}.

Now we define

$$D = 1 + \sum_{\substack{jkm \\ j,k \neq h}} R_{kj,m} [\exp(W_{hkm}^{(0)} - W_{hkm}^{(1)})] \tag{17a}$$

and $$L_{im} = \frac{1}{\sum_{j} W_{ijm}^{(1)}} ; i \neq \text{hom}\, e , \tag{17b}$$

where $R_{p,q,m}$ to be the pq element of matrix R_m.

$$N_{hjm} = [\exp W_{hjm}^{(0)} + \sum_{\substack{pqr \\ p,q \neq h}} R_{pq,r} \exp(W_{hpr}^{(0)} + W_{hjm}^{(1)}) - \sum R_{pq,r} \exp(W_{hpr}^{(1)} + W_{hjm}^{(0)}] / D \tag{18}$$

and

$$N_{ijm} = L_{im}(\exp W_{ijm}^{(1)})\sum_{k\neq h} N_{hkm}R_{ki,m}; i \neq h. \qquad (19)$$

Equations (17a),(18),(19) can be simplified greatly if it is assumed that the utility function $W_{ijm}^{(r)}$ can be written in the form [1] & [4]

$$W_{ijm}^{(r)} = F_{ijm}(x,s,z) + H_r^*(s); i, j \neq h$$

$$= F_{hjm}(x,s,z) + G_r(s); i = h \qquad (20)$$

$$= F_{ihm}(x,s,z) + H_r(s); j = h$$

The time integrated utility function Wcan be represented as the depends on transportation levels of service, household characteristics and destination characteristics but that independent of past travel decisions and future travel plans and a component G or H that represents the effect on time-integrated utility of current travel of non-work trips that have been or will be made at other times of day. Because travel behavior depends only on the difference between the utilities of travel options, there is no loss of generality in assuming that $H_r^*(s)$ is zero for all r and s.

Using the utility specification (20) equation (17a), (18) and (19) become

$$D = 1 + [\exp G_0(s) - \exp G_1(s)] \sum_{\substack{jkm \\ j.k\neq m}} R_{kj,m} \exp F_{hkm} \qquad (21)$$

$$N_{hjm} = D^{-1} \exp[F_{hjm} + G_0(s)] \qquad (22)$$

$$N_{hjm} = D^{-1} L_{im}(\exp F_{ijm}) \sum R_{ki,m} \exp[F_{hkm} + G_0(s)]; i, j \neq h \qquad (23)$$

$$N_{ihm} = D^{-1} L_{im}\{\exp[F_{ihm} + H_1(s)]\} \sum R_{ki,m} \exp[F_{hkm} + G_0(s)]; i \neq h. \qquad (24)$$

In addition, thetotal no. of sojourns at non-work locations, which can be obtained by summation from equation (22) and (23), is

$$N = D^{-1} \sum_{\substack{jkm \\ j,k\neq h}} R_{kj,m} \exp[F_{hkm} + G_0(s) \qquad (25)$$

Equation (22) to (25) together with the definitional relations (17b) and (21) constitute the desired model of non-work travel demand.

Now we have to optimize (20) subject to constraint equations (24) and (25). [1,2]

2. [B]. Maximization of utility:

Now we maximize

$$W_{ijm}^{(r)} = F_{ijm}(x,s,z); i, j \neq h$$

$$= F_{hjm}(x,s,z) + G_r(s); i = h \quad (26)$$

$$= F_{ihm}(x,s,z) + H_r(s);\ j = h \ \ (H_r^*(s) = 0 \forall r \text{ and } s)$$

Subject to the constraint

$$N_{ihm} = D^{-1} L_{im} \{\exp[F_{ihm} + H_1(s)]\} \sum R_{ki,m} \exp[F_{hkm} + G_0(s)]; i \neq h$$

And $$N = D^{-1} \sum_{\substack{jkm \\ j,k \neq h}} R_{kj,m} \exp[F_{hkm} + G_0(s)\,] \quad (27)$$

We use entropy maximizing method to optimize the utility

The Lagrangian of the above model is as follows: [5]

$$\text{L}(x,s,z,\lambda_1,\lambda_2);(i,j \neq h) \quad = \{F_{ijm}(x,s,z)$$

$$+\lambda_1 (N_{ihm} - D^{-1} L_{im} \{\exp[F_{ihm} + H_1(s)]\} \sum R_{ki,m} \exp[F_{hkm} + G_0(s)]; i \neq h)$$

$$+\lambda_2 (N - D^{-1} \sum_{\substack{jkm \\ j,k \neq h}} R_{kj,m} \exp[F_{hkm} + G_0(s))\} \quad \text{Now } \frac{\partial L}{\partial x} = 0$$

Gives the value of N_{ihm}

$$\frac{\partial}{\partial x}(N_{ihm}) = \{(L_{im} \exp F_{ihm} \frac{\partial D^{-1}}{\partial x} + D^{-1} \exp F_{ihm} \frac{\partial}{\partial x} L_{im} + D^{-1} L_{im} \frac{\partial}{\partial x} \exp F_{ihm})$$
$$(\exp F_{hkm} \frac{\partial}{\partial x} \sum_{k \neq h} R_{kim} + \sum_{k \neq h} R_{kim} \frac{\partial}{\partial x} \exp F_{ihm})\} + \frac{\lambda_2}{\lambda_1}\{-\frac{\partial N}{\partial x} + \exp F_{hkm} \frac{\partial D^{-1}}{\partial x} \sum_{jkm} R_{kjm} + D^{-1} \exp F_{hkm} \frac{\partial}{\partial x} \sum_{jkm} R_{kjm}$$
$$+ D^{-1} \sum_{jkm} R_{kjm} \frac{\partial}{\partial x} \exp F_{hkm}\} - \frac{\partial}{\partial x} F_{ijm}(x,s,z) \quad (28)$$

and $\frac{\partial L}{\partial s} = 0$ gives the value of L_{im}

$$\lambda_1 [\{\sum_{k \neq h} R_{kim} \frac{\partial}{\partial s} \exp F_{hkm}\}\{L_{im} \exp F_{ihm} \frac{\partial}{\partial s} D^{-1} + \exp F_{ihm} D^{-1} \frac{\partial L_{im}}{\partial s} + L_{im} D^{-1} \frac{\partial}{\partial s} \exp F_{ihm}\}]$$
$$= -\lambda_1 [\frac{\partial}{\partial s} N_{ihm}] - [\frac{\partial}{\partial s} F_{ijm}(x,s,z)]$$
$$-\lambda_2 [\frac{\partial N}{\partial s} - \{\exp F_{hkm} \sum_{jkm} R_{kjm} \frac{\partial D^{-1}}{\partial s} + D^{-1} \exp F_{hkm} \frac{\partial}{\partial s} \sum_{jkm} R_{kjm} + D^{-} \sum_{khm} R_{hkm} \frac{\partial \exp F_{hkm}}{\partial s}] \quad (29)$$

Also from $\frac{\partial L}{\partial z}=0$ gives

$$\frac{\partial N}{\partial z}=[\exp F_{hkm}\sum_{jkm}R_{kjm}\frac{\partial D^{-1}}{\partial z}+D^{-1}\exp F_{hkm}\frac{\partial}{\partial z}\sum_{jkm}R_{kjm}+D^{-1}\sum_{jkm}R_{kjm}\frac{\partial \exp F_{hkm}}{\partial z}]-\frac{\partial}{\partial z}F(x,s,z)$$

$$-\frac{\lambda_1}{\lambda_2}[\frac{\partial N_{ihm}}{\partial z}-\{L_{im}\exp F_{ihm}\frac{\partial D^{-1}}{\partial z}+\exp F_{ihm}D^{-1}\frac{\partial L_{im}}{\partial z}+L_{im}D^{-1}\frac{\partial \exp F_{ihm}}{\partial z}\}\{\sum_{k\neq h}R_{hkm}\frac{\partial F_{hkm}}{\partial z}+F_{hkm}\frac{\partial}{\partial z}\sum_{k\neq h}R_{hkm}\}]$$

…….(30)

Since G and H is independent of past travel decision, then three cases of time integrated utility function (26) can be expressed by the first one.
Thus from the above equations (28), (29) and (30) we get the values of N_{ihm} , L_{im},N which optimizes the utility function W_{ijm} .[1,2,4]

3. Conclusion:

In this paper we optimize the utility function using entropy optimization method instead of utility maximizing method. Now considerable attention has been paid in the transportation research literature to the interpretation of transportation levels of service, household characteristics and destination characteristics from the application of above model. Since this objective function is intimately related to entropy maximization, some general conclusions are drawn on the relation between the properties of model based on entropy maximization.

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