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Research Paper

A Decomposition Algorithm for Solving Chance Constrain Bi Level Multi Objective Large Scale Quadratic Programming Problem

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Abstract: *This paper solves a bi-level multi objective large scale quadratic programming problem with stochastic parameters in the constraints (SBLMOLSQPP). We solve this problem usingan algorithm that begins with transforming the probabilistic nature to equivalent deterministic of this problem, thenTaylor series is used to overcome the complexity of the quadratic problem. Finally, an illustrative numerical example is given to clarify the developed theory.*

Keywords: Chance constraint, bi level, multi objective, large scale problems, quadratic programming.

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1. Introduction

Decision problems of stochastic or probabilistic optimization arise when certain coefficients of an optimization model are random quantities. Stochastic multi objective programs are challenging from both computational and theoretical points of view since they combine three different types of models into one. Until now algorithmic results have been limited to special instances [1].

Stochastic or chance constraint programming is a mathematical programing where some of the data in the objective function or in the constraints are uncertain. Uncertainty is usually represented by a probability distribution on the parameters.

Bi level programming problems (BLPPs) are hierarchical optimization problems in which there exist two decision makers (DMs) who have different priorities on decision. It is assumed that the DM at the upper level, who has higher priority than the other, first specifies a strategy, and then the DM at the lower level chooses a strategy so as to optimize its own objective with full knowledge of the action of the DM at the upper level [2].

Multi-objective optimization problems are a class of difficult optimization problems in which several different objective functions have to be considered simultaneously. Usually, there is no solution optimizing simultaneously all the several objective functions [3].

Large scale programming which closely describes and represents the real world decision situations, various factors of the real system should be reflected in the description of the objective function and constraints, naturally these objective function and constraints involve many parameters and the experts may assign them different values [4, 5].

Emam et al. [6] Presented an algorithm to solve bi-level multi-objective large scale quadratic programming problem with stochastic parameter in the objective functions. The objective of the first phase of the solution algorithm is to avoid the complexity of the stochastic nature and converts the problem into crisp problem, then Taylor series is combined with weight method to convert the multi-objective quadratic problem into linear objective function. Therefore, the decomposition algorithm is usedto get the optimal solution for this problem. Emam et al. [7] solved bi-level large scale quadratic programming problem with stochastic parameters in the constraints (SBLLSQPP).

Alrefaei et al. [8] addressed the multi-objective stochastic optimization problem that arises in many real-world applications, especially in supply chain management and optimization. To this end, a simulated annealing algorithm is presented and used for solving this problem. The algorithm uses the hill-climbing criterion in order to escape from local minimalist trap. The paper also introduces a new Pareto set for stochastic optimization problems and demonstrates the application of simulated annealing on this Pareto set.

Lachhwani [9] proposed an alternate technique based on fuzzy goal programming approach for solving multi-level multi objective linear programming problem (ML-MOLPP).In formulation of FGP model each objective functions at each level are transformed into fuzzy goals. Suitable membership function for every fuzzily described transformed objective functions at each level as well as the control vectors of each level decision makers are defined by determining individual optimal solution of each objective function at each of the decision making level. Then FGP approach is used for achieving highest degree of each of these membership goals by minimizing the sum of negative deviational variables.

Currently the major challenging task for this paper is how to solve large scale bi level multi objective problem with probabilistic nature in constrains, so this paper introduces an algorithm to solve this problem.

The rest of the paper is organized as follows: we start in Section 2 by formulating the model of a bi-level multi-objective large scale quadratic programming problem with stochastic parameters in constrains. Section 3 converts the stochastic in constrains into deterministic. Section 4 presents a Taylor series approach for bi-level large scale multi-objective quadratic programming problem (BLSLMOQPP) to convert the quadratic objective functions to linear objective functions. In section 5, the decomposition method for large scale bi-level linear programming problem is presented. In Section 6, an example is provided to describe the developed results. Finally, Section 7 concludes the paper and states some open points for future research work in the area of stochastic bi-level multi-objective quadratic programming optimization problems.

2. Problem Formulation and Solution Concept

The bi-level large scale multi-objective quadratic programming problem (BLLSMOQPP) with stochastic parameters in constrains may be formulated as follows:

[Upper Level]

$$\underset{x_1,x_2}{Max}\ F_1(x) = \underset{x_1,x_2}{Max}\ (f_{11}(x), f_{12}(x), \ldots, f_{1u}(x)), \tag{1}$$

Where $x_3, x_4, \ldots, x_m$ solves

[Lower Level]

$$\underset{x_3,x_4}{Max}\ F_2(x) = \underset{x_3,x_4}{Max}(f_{21}(x), f_{22}(x), \ldots, f_{2n}(x)), \tag{2}$$

Subject to

$$x \in G. \tag{3}$$

Where

$$G = \{ pr(a_{01}x_1 + a_{02}x_2 + a_{03}x_3 + a_{04}x_4 + a_m x_m \le b_0) \ge \alpha_0,$$
$$pr(d_1 x_1 \le b_1) \ge \alpha_1,$$
$$pr(d_2 x_2 \le b_2) \ge \alpha_2,$$
$$pr(d_3 x_3 \le b_3) \ge \alpha_3,$$
$$pr(d_4 x_4 \le b_4) \ge \alpha_4,$$
$$pr(d_m x_m \le b_m) \ge \alpha_m$$
$$x_1, \ldots, x_m \ge 0. \}$$

And also where

$$f_{ij} = c_{ij}x + \frac{1}{2}x^T L_j^i\ x,\ (i = 1, 2),\ (j = 1, 2, \ldots, n_i) \tag{4}$$

Let the functions F_1 and F_2 are quadratic objective functions defined on R^n.

Let x_1, x_2, x_3, x_4 be real vector variables indicating the first decision level's choice and the second decision level's choice. Moreover, the upper level decision maker has x_1, x_2 indicating the first decision level choice, the lower level decision maker have x_3, x_4 indicating the second decision level choice.

Let (L^1, L^2) are $m \times n$ matrices describing the coefficients of the quadratic terms and c_{ij} are $1 \times m$ matrices. In the above problem (1) – (4), x is m real vector variables. Let G be the large scale linear constraint set where, $b = (b_0, \ldots, b_m)^T$ is $m + 1$ vector, and $a_{01}, \ldots, a_{04}, d_1, \ldots, d_4$ are constants.

Furthermore p stands for probability and α_i is a specified probability value.

This means that the linear constraints may be violated some of the time and at most 100(1-α_i) % of the time. For the sake of simplicity, we assume that the random parameters b_i, (i =1, 2,…, m) are distributed normally with known means E{ b_i } and variances V{ b_i }and independently of each other.

Definition 1: *For any* $(x_1, x_2 \in G_1 = \{x_1, x_2 | (x_1, .x_2, x_3, ..., x_m) \in G\})$ *given by upper level, if the decision-making variable* $(x_3, x_4 \in G_2 = \{x_3, x_4 | (x_1, .x_2, x_3, ..., x_m) \in G\})$ *is the Pareto optimal solution of the lower level, then* $(x_1, .x_2, x_3, x_4)$ *is a feasible solution of (BLLSMOQPP).*

Definition 2: *If* $x^* \in R^m$ *is a feasible solution of the (BLMOLSQPP) with probability* $\prod_{i=1}^{m} \alpha_i$ *; no other feasible solution* $x \in G$ *exists, such that* $F_1(x^*) \le F_1(x)$ *so* x^* *is the Pareto optimal solution of the (BLMOLSQPP).*

3. Stochastic Transformation

The basic idea is to convert the probabilistic nature of stochastic bi-level large scale multi-objective programming problem into deterministic problem by using [10]:

$$X' = \left\{ X \in R^n \middle| \sum_{j=1}^{n} a_{ij} x_j \le E(b_i) + K_{\alpha_i} \sqrt{Var\ (b_i)}, (i = 1,2,...., m), x_j \ge 0, (j = 1,2,.... m) \right\} \tag{5}$$

Where K_{α_i} is the standard normal value such that $\Phi(K_{\alpha_i}) = 1 - \alpha_i$; and $\Phi(a)$ represents the "cumulative distribution function" of the standard normal distribution evaluated at a.

Then the problem can be understood as the corresponding deterministic bi-level large scale multi-objective quadratic programming problem as following:

[Upper Level]

$$\underset{x_1, x_2}{Max} \quad F_1(x) = \underset{x_1, x_2}{Max} \; [f_{11}(x), ..., \quad f_{1U}(x)] \tag{6}$$

[Lower Level]

$$\underset{x_3, x_4}{Max} \quad F_2(x) = \underset{x_1, x_2}{Max} \; [f_{21}(x), ..., \quad f_{1n}(x)] \;, \tag{7}$$

$$x \in G'. \tag{8}$$

Where

$$\begin{aligned} G' = \{\, & a_{01}x_1 + a_{02}x_2 + a_{04}x_4 \le b_0, \\ & d_1 x_1 \le b_1, \\ & d_2 x_2 \le b_2, \\ & d_4 x_4 \le b_m, \\ & x_1, ..., x_4 \ge 0.\} \end{aligned}$$

4. Taylor Series Approach

We use the weighting method to transform the objective functions in the upper level and lower level from multi-objective into single objective.

To solve a bi level large scale quadratic programming problem using decomposition algorithms is a complex problem. Taylor series can overcome this complexity by obtaining polynomial objective functions equivalent to quadratic objective functions in the following form [11]:

$$K_i(x) \cong F_i^{\wedge}(x) = F_i(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial F_i(x_j^*)}{dx_j}, (j = 1,2,..m), (i = 1,2) \quad (9)$$

So the BLLSPP can be written as:

[Upper Level]

$$\underset{x_1, x_2}{Max} \quad K_1(x\ ), \quad (10)$$

Where x_3, x_4 solves

[Lower Level]

$$\underset{x_3, x_4}{Max} \ K_2(x), \quad (11)$$

Subject to

$$x \in G'. \quad (12)$$

5. Decomposition Algorithm for Bi Level Large Scale Linear Programming Problem

The bi level large scale linear programming problem is solved by adopting the leader-follower Stackelberg strategy combined with Dantzig and Wolf decomposition method [12]. First, the optimal solution that is acceptable to the FLDM is obtained using the decomposition method to break the large scale problem into n-sub problems that can be solved directly.

The decomposition technique depends on representing the BLLSLPP in terms of the extreme points of the sets $d_s x_s \le b_s$, $x_s \ge 0$, $s = 1,2,..,m$.To do so, the solution space described by each $d_s x_s \le b_s$, $x_s \ge 0$, $j = 1,2,..,m$ must be bounded and closed.

After that by inserting the upper level decision variable to the lower level for him/her to search for the optimal solution using Dantzig and Wolf decomposition method [12], then the decomposition method breaks the large scale problem into n-sub problems that can be solved directly and obtain the optimal solution for his/her problem which is the optimal solution to the BILSPP.

Theorem 1: *The decomposition algorithm terminates in a finite number of iterations, yielding a solution of the large scale problem.*

To prove theorem 1 above, the reader can refer to [12].

6. An Algorithm for Solving BLLSMOQP Problem

6.1 Algorithm

This is an algorithm to solve bi-level large scale multi-objective quadratic programming problem programming problem (BLLSMOQPP) with stochastic parameters in constrains.

This algorithm conquers the complexity nature of the bi level large scale quadratic programming problem. The suggested algorithm can be summarized in the following manner:

Step 1: Calculate the $E\{\theta_j^r\} and var\{\theta_j^r\}$.

Step 2: Convert from stochastic to deterministic formula.

Step 3: Formulate the equivalent bi-level large scale multi-objective quadratic programming

Step 4: Use weight method to convert from multi objective to single objective.

Step 5: Convert bi level large scale multi-objective quadratic programming to linear by using Taylor series approach.

Step 6: Start with the upper level problem and convert the master problem in terms of extreme points of the sets $d_s x_s \le b_s, x_s \ge 0, s = 1, 2, 3$.

Step 7: Determine the extreme points $x_s = \sum_{k=1}^{k_j} \beta_{sk} \hat{x}_{sk.}, s = 1,2,3$ using Balinski's algorithm [13].

Step 8: Set $k = 1$.

Step 9: Compute $z_{sk} - c_{sk} = C_B B^{-1} P_{sk} - c_{sk}$.

Step 10: If $\overset{*}{z}_{sk} - \overset{*}{c}_{sk} \le 0$, then go to Step 11; otherwise, the optimal solution has been reached, go to Step 16.

Step 11: Determine $\hat{X}_{sk}$ associated with $\min\{\overset{*}{z}_{sk} - \overset{*}{c}_{sk}\}$

Step 12: β_{jk} associated with extreme point $\hat{X}_{sk}$ must enter the solution

Step 13: Determine the leaving variable

Step 14: The new basis is determined by replacing the vector associated with leaving variable with the vector β_{sk}.

Step 15: Set $k = k + 1$, go to step 9.

Step 16: If the SLDM obtained the optimal solution go to Step 19, otherwise go to Step 17

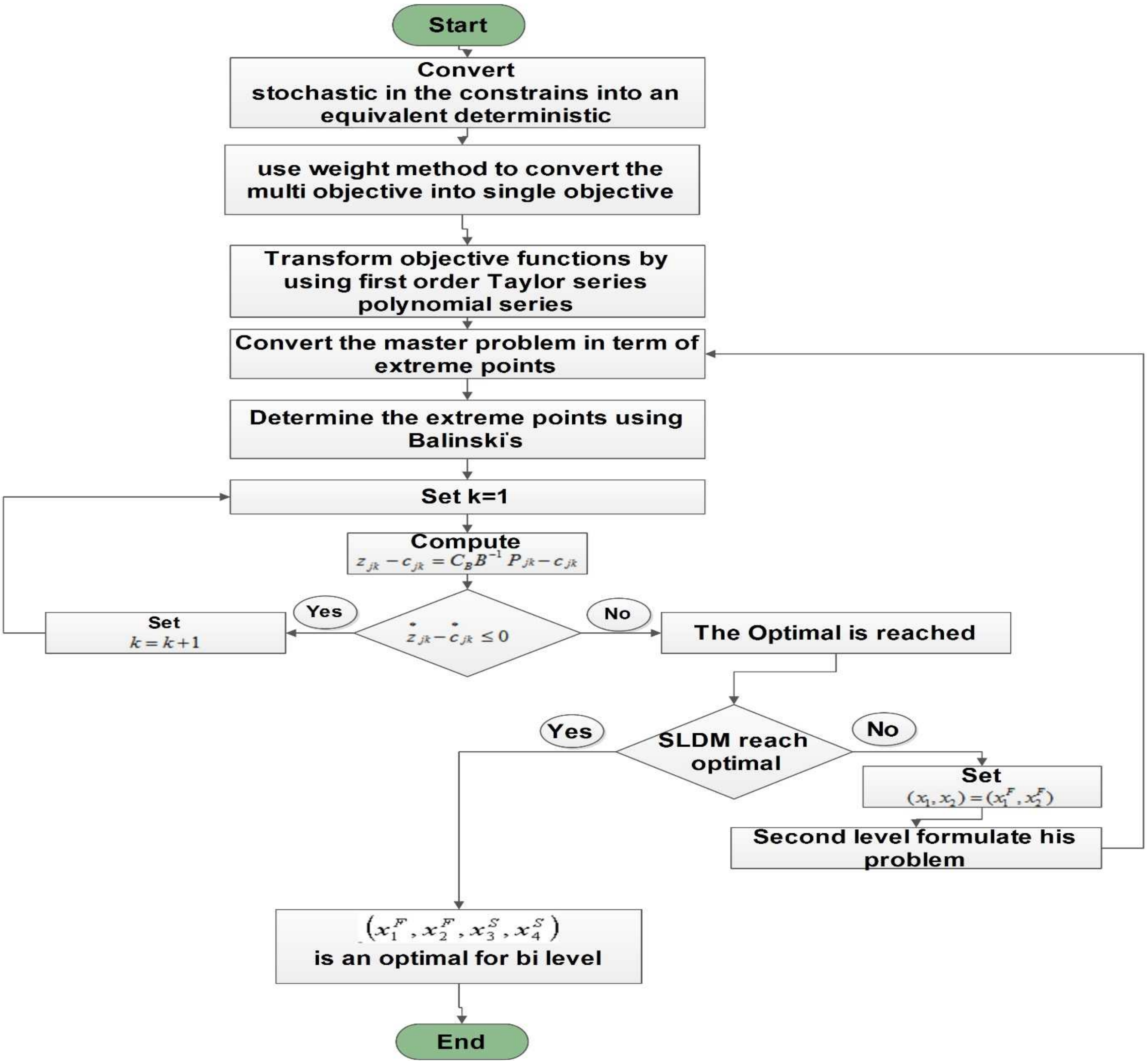
Step 17: Set $(x_1, x_2) = (x_1^F, x_2^F)$ to the SLDM constraints.

Step 18: The SLDM formulate his problem, go to Step 8.

Step 19: $(x_1^F, x_2^F, x_3^S, x_4^S)$Is as an optimal solution for bi-level large scale linear programming problem, then stop.

6.2 A Flowchart

A flowchart to explain the suggested algorithm for solving BLLSMOQPP is described as follows:



7. Numerical Example

The solution for (SBLMOLSQPP) is considered as:

[Upper Level]

$$\underset{x_1,x_2}{Max}\ F_1(x) = \underset{x_1,x_2}{Max}(6x_1^2 + 2x_2^2 + 2x_3 + 2x_4\,,\ 4x_1^2 + 8x_2^2 + 6x_4)$$

Where x_3, x_4 solves

[Lower level]

$$\underset{x_3,x_4}{Max}\ F_2\left(x\right)=\underset{x_3,x_4}{Max}(2\,x_1\ +2x_3^2\ +2x_4^2,\ 4x_2\ +2x_3^2\ +6x_4^2)$$ 20\

Subject to:

$$pr(x_1+x_2+x_3+x_4\le v_1)\ge 0.0007,$$
$$pr(4x_1+x_2\le v_2)\ge 0.0228,$$
$$pr(4x_3+2x_4\le v_3)\ge 0.0026,$$
$$pr(x_3+4x_4\le v_4)\ge 0.0013,$$
$$x_1,x_2,x_3,x_4\ge 0.$$

Suppose that:
v_i (i =1, 2, 3, 4) has independent normal distribution with the following means and variances

Table 1: Means and variances variables

Mean	**Variance**
E(v_1)=10	Var(v_1)=225,
E(v_2)=8 ,	Var(v_2)=256
E(v_3)=5	Var(v_3)=100
E(v_4)=8	Var(v_4)=144

Now the (SBLMOLSQPP) with stochastic parameters in constraints can be understood as following deterministic bi level large scale quadratic programming problem (BLLSQPP):

[Upper level]

$$\underset{x_1,x_2}{Max}\ F_1\left(x\right)=\underset{x_1,x_2}{Max}(6x_1^2+2x_2^2+2x_3+2x_4,\ 4x_1^2+8x_2^2+6x_4)\,,$$

Where x_3,x_4 solves

[Lower level]

$$\underset{x_3,x_4}{Max}\ F_2\left(x\right)=\underset{x_3,x_4}{Max}(2\,x_1\ +2x_3^2\ +2x_4^2,\ 4x_2\ +2x_3^2\ +6x_4^2)\,,$$

Subject to

$$x_1+x_2+x_3+x_4\le 58,$$
$$4x_1+x_2\le 40\,,$$
$$4x_3+2x_4\le 32,$$
$$x_3+4x_4\le 30\,,$$
$$x_1,\,x_2,\,x_3,\,x_4\ \ \ge 0$$

Using the weighting method, then the problem (BLMOQPP) can be written with a single-objective function in upper level and lower level as following:

[Upper level]

$$\underset{x_1,x_2}{Max}\ H_1(x) = \underset{x_1,x_2}{Max}(5x_1^2 + 5x_2^2 + x_3 + 4x_4)\ ,$$

[Lower level]

$$\underset{x_3,x_4}{Max}\ H_2(x) = \underset{x_3,x_4}{Max}\ (x_1 + 2x_2 + 2x_3^2 + 4x_4^2)\ ,$$

Subject to

$$x_1 + x_2 + x_3 + x_4 \le 58,$$
$$4x_1 + x_2 \le 40,$$
$$4x_3 + 2x_4 \le 32,$$
$$x_3 + 4x_4 \le 30,$$
$$x_1, x_2, x_3, x_4 \ \ge 0.$$

Using the first order Taylor polynomial series to convert the quadratic function to linear function. Therefore, the (BLLSPP) is written as:

[Upper level]

$$\underset{x_1,x_2}{Max}\ K_1(x) = \underset{x_1,x_2}{Max}(20x_1 + 10x_2 + x_3 + 4x_4 - 25)\ ,$$

[Lower level]

$$\underset{x_3,x_4}{Max}\ K_2(x) = \underset{x_3,x_4}{Max}(x_1 + 2x_2 + 12x_3 + 4x_4 - 18)\ ,$$

Subject to

$$x_1 + x_2 + x_3 + x_4 \le 58,$$
$$4x_1 + x_2 \le 40,$$
$$4x_3 + 2x_4 \le 32,$$
$$x_3 + 4x_4 \le 30,$$
$$x_1, x_2, x_3, x_4 \ \ge 0.$$

The FLDM problem is formulated as follows:

$$\underset{x_1,x_2}{Max}\ K_1(x) = \underset{x_1,x_2}{Max}(20x_1 + 10x_2 + x_3 + 4x_4 - 25)$$

Subject to

$$x_1 + x_2 + x_3 + x_4 \le 58,$$

$4x_1 + x_2 \leq 40,$
$x_1, x_2, x_3, x_4 \geq 0.$

After four iterations the FLDM obtains the optimal solution.

$\left(x_1^F, x_2^F, x_3^F, x_4^F\right) = (0,40,0,7.5).$

So, $K_1 = 405.$

Now set $(x_1, x_2) = (0,40)$ to the SLDM constraints.

Secondly, the lower level solves the problem as follows:

$$\underset{x_3, x_4}{Max}\ K_2(x) = \underset{x_3, x_4}{Max}(12x_3 + 4x_4 + 62)$$

Subject to

$x_3 + x_4 \leq 18,$
$4x_3 + 2x_4 \leq 32,$
$x_3 + 4x_4 \leq 30,$
$x_3, x_4 \geq 0.$

The second level decision maker will repeat the same steps as the first level decision maker until the second level decision maker gets the optimal solution so:

$\left(x_3^s, x_4^s\right) = (8,0).$

So, $\left(x_1^s, x_2^s, x_3^s, x_4^s\right) = (0,40,8,0).$

$K_2 = 158.$

8. Summary and Concluding Remarks

This paper presented a bi level large scale multi objective quadratic programming problem with stochastic parameters in constrains. The paper introduces a powerful algorithm to solve (BLSLMOQPP).Firstly the probabilistic nature of the problem is converted intoan equivalent crisp problem. Then Taylor series is used to convert the quadratic problem into linear problem to be easy for solving with the decomposition algorithm. Finally, the numerical example was introduced the result of this paper.

However, there are many other aspects, which should be explored and studied in the area of a large scale bi-level optimization such as:

1. Large scale bi-level fractional programming problem with rough parameters in the objective functions, in the constraints and with integrality conditions.
2. Large scale multi-level non-linear programming problem with stochastic parameters in the constraints and with integrality conditions.

3. Large scale multi-level non-linear programming problem with stochastic parameters in the objective functions, in the constraints and with integrality conditions.

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