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Research Paper

MHD Free Convection Reacting Variable Viscosity Power-Law Flow on a Porous Plate

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Abstract: *We considered the flow of a viscous fluid flowing in a two dimensional channel between two parallel plates placed at y=h and y=-h. The momentum equation considered was non-dimensionalised after the introduction of buoyancy and porosity term. We are able to showed that the solution exist and unique. The effects of introduction terms were displayed graphically.*

Keywords: Free convection, Variable viscosity, Power law, Porous plate, two dimensional channel, Parallel plate, Buoyancy, Porosity.

1.0 Introduction

If a liquid or gas is heated, its mass per unit volume generally decreases. But if the liquid or gas is in gravitational field, the hotter, lighter fluid rises while the colder, heavier fluid sinks. This kind of motion, due solely to non-uniformity of fluid temperature in the presence of a gravitational field is called *free or natural convection*. It's also responsible for the rising of the hot water and steam in natural- convection boiler and for the draft in a chimney [3]. Prasada et al. [6] studied combined effect of free and forced convection on MHD flow in a rotating porous channel. They gave a steady linear theory of the combined effect of the free and forced convection in rotating hydro magnetic viscous fluid flow in a porous channel under the action of a uniform magnetic field. The flow is governed by the Grashof number *G*, the Hartmann number *H*, the Ekman *E* and the suction Reynolds number *S*. The solution for the velocity field, temperature distribution, magnetic field, mass flow rate and the shear stresses on the channel boundaries were obtained using a perturbation method with the small parameter S. The nature of the associated boundary layers was investigated for various values of the governing flow parameters. Also, the velocity, temperature and shear stresses were discussed numerically by drawing profiles with reference to the variations in the flow parameters. EL-Kabeir et al. [5] worked on unsteady MHD combined convection over a moving vertical sheet in a flux saturated porous medium with uniform surface heat flux in which group transformation method is applied for solving the combined convection problem in an unsteady, two-dimensional, laminar, boundary-layer flow of a viscous incompressible and electrical-conducting fluid along a vertical, continuous moving plate, saturated, porous medium in the presence of a uniform transverse magnetic field in which discussion were provided for the effect of magnetic parameter M, permeability of the porous medium k and Prandtl number Pr on the velocity, temperature fields within the boundary layer, shear stress and heat transfer. Rahman et al [7]: MHD natural convection flow of an electrically conducting fluid along a vertical flat plate with temperature dependent thermal conductivity effects is analyzed. The governing equations with associated boundary condition for the phenomenon were converted to dimensionless form using a suitable transformation. The transformed non-linear equations were solved using implicit finite difference method with Keller-box scheme. Numerical results of the velocity, temperature, skin friction coefficient and surface temperature for different values of the magnetic parameter, thermal conductivity variation parameter, Prandtl number and conjugate conduction parameter were also presented graphically. Ravikumar V et al [8] studied a two dimensional steady free convective and mass transfer flow of an electrically conducting, viscous fluid through a porous medium bounded by two stationary infinite vertical porous plates in the presence of thermo diffusion and chemical effect.

Eldabe N.M.T et al. [4] studied the effect of couple stresses on the MHD of a Non-Newtonian unsteady flow between two parallel porous plates in which the MHD of a Non-Newtonian unsteady flow of an incompressible fluid under the effect of couple stresses and a uniform external magnetic field is analyzed by using the Eyring Powell model. They considered two cases, In the first case, the solution was obtained by using the Mathematical computational program which assumed a pulsatile gradient in the direction of the motion. In the second case, a numerical solution of the non-linear partial differential equation was obtained by using a finite difference method.

2.0 Mathematical Formulation

Following [4], the governing equation is

$$\rho\left(\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\sigma B_0^{\;2}u-\eta\frac{\partial^4 u}{\partial y^4} \tag{2.1}$$

Subject to boundary conditions:

$$u=u_1=0, u''=u_1''=0 \;\; at \;\; y=0 \tag{2.2}$$

$$u=u_1=0, u''=u_1''=0 \;\; at \;\; y=1 \tag{2.3}$$

For the purpose of this work, we introduce buoyancy and porosity term to (2.1) to have

$$\rho(\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+\rho g\beta(T-T_0)-\sigma B_0^{\;2}u-\eta\frac{\partial^4 u}{\partial y^4}-\frac{\mu}{D}u \tag{2.4}$$

Where

$\frac{\partial u}{\partial t}$ is the rate of change of velocity with time, $\frac{\partial p}{\partial x}$ is the pressure gradient along $x-$component, $\frac{\partial u}{\partial y}$ is the velocity gradient along $y-$component, μ is the dynamic viscosity, $u(h)$ is the velocity at the upper plate of channel, $u(-h)$ is the velocity at the lower plate of channel, $\frac{\partial}{\partial y}$ is the partial derivative with respect to position p – pressure, ρ is the density of the fluid, v_0 is the suction velocity, $u(y)$ is the maximum velocity, g is acceleration due to gravity, x is the horizontal coordinate, u is the horizontal velocity component, β is the coefficient of thermal expansion, k is the thermal conductivity, D is the permeability constant, T is the temperature, T_0 is the initial temperature, η is the coefficient of couple stresses, B_0 is the external magnetic field, σ is electric conductivity of the fluid.

Further assumption includes; suction velocity and couple stresses were neglected, then (2.4) becomes:

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+\rho g\beta(T-T_0)-\sigma B_0{}^2u-\frac{\mu}{D}u \tag{2.5}$$

With initial and boundary conditions:

$$u(-L)=u(L)=0 \tag{2.6}$$

We assume that the pressure gradient and dynamic viscosity are:

$$-\frac{\partial p}{\partial x}=c[\gamma+(1-({}^{y'}\!/\!_{h})^2)] \tag{2.7}$$

$$\mu=\mu_0\exp[\gamma+(1-({}^{y'}\!/\!_{h})^2+\beta\frac{E(T-T_0)}{RT_0})]\left(\frac{\partial u}{\partial y'}\right)^n \tag{2.8}$$

Where, c is a constant, y' is the vertical coordinate, μ_0 is viscosity at $y'=h$, h is the gap between parallel plates, E is the activation energy, R is the universal gas constant, γ is a constant, n is the power law index,

Substituting (2.7) and (2.8) into (2.5), we obtain

$$\begin{aligned}\rho\frac{\partial u}{\partial t}=&-(c[\gamma+(1-({}^{y'}\!/\!_{h})^2])+\frac{\partial}{\partial y}((\mu_0\exp[\gamma+(1-{}^{y'}\!/\!_{h})^2+\\&\beta\frac{E(T-T_0)}{RT_0})]((\frac{\partial u}{\partial y'})^2)\frac{\partial u}{\partial y})+\rho g\beta(T-T_0)-\sigma B_0{}^2u-\frac{\mu}{D}u\end{aligned} \tag{2.9}$$

We non-dimensionalized equation (2.9) using the following parameters

$$\phi=\frac{u}{u_0},\ y'=\frac{y}{L},\ T=T_0+\frac{RT_0\theta}{E},\ \varepsilon=\frac{RT_0}{E} \tag{2.10}$$

After dropping prime, as $y\rightarrow 0$ and $\frac{\partial}{\partial t}=0$, (2.9) becomes:

$$\frac{d}{dy}(\mu_0\exp[\gamma+1+\beta_1\theta]((\frac{d\phi}{dy})^n)+G\theta-(Ha^2-A\exp^{\gamma+1+\beta_1\theta})\phi-F=0 \tag{2.11}$$

$$\phi(-1)=\phi(1)=0 \tag{2.12}$$

Where n=1, $G=\dfrac{L^2\rho g\varepsilon\beta_1}{u_0}$, $Ha=LB_0\sqrt{\sigma}$, $A=\dfrac{L^2\mu_0}{D}$, $F=\dfrac{-cL^2}{u_0}(\gamma+1)$ (2.13)

Here, G is Grashof number, Ha is Hartmann number, A is Porosity term, F is Body force. From (2.11), we assume that its line true from -1 to 1. $y\in(-\varepsilon,\ \varepsilon)$, near $y=0,\ \theta=\theta_{\max}$ (i.e. θ is constant).

Now (2.11) becomes:

$$\frac{d^2\phi}{dy^2}+\frac{G\theta_{\max}+F}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}-\frac{(Ha^2+Ae^{\gamma+1+\beta\theta_{\max}})\phi}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}=0 \tag{2.14}$$

$$\phi(-1)=\phi(1)=0 \tag{2.15}$$

3.0 Existence and Uniqueness of Solution

Theorem: *Suppose* $-1\le y\le 1$ *where* $\mu_0,\ \gamma,\ \beta,\ F,\ Ha^2,\ A,\ G,\ \theta_{\max}$ *and* a *are real constant, then the problem (2.14) satisfies (2.15) has a unique solution.*

Proof: Let

$$x_1=y \quad x_2=\phi \quad x_3=\phi'$$

Then

$$\phi''=\frac{(Ha^2+Ae^{\gamma+1+\beta\theta_{\max}})x_2}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}-\frac{G\theta_{\max}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}-\frac{F}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}$$

The system of equations can be written in vector form using

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}y\\\phi\\\phi'\end{pmatrix}$$

As

$$\begin{pmatrix}x_1'\\x_2'\\x_3'\end{pmatrix}=\begin{pmatrix}1\\x_3\\\dfrac{(Ha^2+Ae^{\gamma+1+\beta\theta_{\max}})x_2}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}-\dfrac{G\theta_{\max}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}-\dfrac{F}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}\end{pmatrix}$$

Satisfying

$$\begin{pmatrix}x_1(-1)\\x_2(-1)\\x_3(-1)\end{pmatrix}=\begin{pmatrix}-1\\0\\a\end{pmatrix}$$

$-1\le y\le 1,$ where a is guessed such that $x_2(1)=0$

We define

$$f_1\ (x_1, x_2, x_3) = 1$$
$$f_2(x_1, x_2, x_3) = x_3$$
$$f_3(x_1, x_2, x_3) = \frac{(Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}})x_2}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}} - \frac{G\theta_{\max}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}} - \frac{F}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}$$

Then

$$\left|\frac{\partial f_1}{\partial x_i}\right| = 0 \quad for\ i = 1,\ 2,\ 3,\ \left|\frac{\partial f_2}{\partial x_i}\right| = 0 \quad for\ i = 1,\ 2,\ \left|\frac{\partial f_2}{\partial x_3}\right| = 1,\ \left|\frac{\partial f_3}{\partial x_i}\right| = 0 \quad for\ i = 1,\ 3$$
$$\left|\frac{\partial f_3}{\partial x_2}\right| = \frac{(Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}})}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}$$

Let $K = \max\left\{1,\ \frac{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}\right\} < \infty$.

The partial derivatives $\frac{\partial f_i}{\partial x_j},\ i,\ j = 1,\ 2,\ 3$ are bounded since there exist a constant $K > 0$ such that $\left|\frac{\partial f_i}{\partial x_j}\right| \le K,\ i,\ j = 1,\ 2,\ 3,$ where K is the Lipchitz. Hence an equation (2.14) satisfying (2.15) has a unique solution.

4.0 Method of Solution

We solve (2.14) subject to (2.15) analytically by finding the complementary and particular solution.

Complementary Solution

$$\frac{d^2\phi}{dy^2} = \frac{(Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}})\phi}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}} \tag{4.1}$$

Let

$$\phi = e^{my},\ \frac{d\phi}{dy} = me^{my},\ \frac{d^2\phi}{dy^2} = m^2 e^{my} \tag{4.2}$$

$$\Rightarrow\ m = \sqrt{\frac{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}} \tag{4.3}$$

Let

$$\phi_c = c_1 e^{my} + c_2 e^{-my} \tag{4.4}$$

Then

$$\phi_c = c_1 e^{\left(\sqrt{\frac{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}}\right)y} + c_2 e^{\left(-\sqrt{\frac{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}}\right)y} \tag{4.5}$$

Particular Solution

$$\frac{d^2\phi}{dy^2} - \frac{(Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}})\phi}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}} = -(G\theta_{\max} + F)\mu_0 e^{-(\gamma+1+\beta\theta_{\max})} \tag{4.6}$$

Since

$-(G\theta_{\max} + F)\mu_0 e^{-(\gamma+1+\beta\theta_{\max})}$ gives constant

We let

$$\phi_p = k, \quad then\ \phi_p'' = 0 \tag{4.7}$$

Equation (4.6) becomes

$$0 - \frac{(Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}})k}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}} = -(G\theta_{\max} + F)\mu_0 e^{-(\gamma+1+\beta\theta_{\max})} \tag{4.8}$$

Therefore,

$$k = \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} = \phi_p \tag{4.9}$$

But

$$\phi = \phi_{complementary} + \phi_{particular} \tag{4.10}$$

$$\therefore\ \phi = c_1 e^{my} + c_2 e^{-my} + \phi_p \tag{4.11}$$

Then

$$\phi = c_1 e^{\left(\sqrt{\frac{Ha^2+Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}}\right)y} + c_2 e^{\left(-\sqrt{\frac{Ha^2+Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}}\right)y} + \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} \tag{4.12}$$

We take

$$\phi(-1) = \phi(1) = 0$$

Then

$$c_1 e^{\left(-\sqrt{\frac{Ha^2+Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}}\right)} + c_2 e^{\left(\sqrt{\frac{Ha^2+Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}}\right)} + \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} = 0 \tag{4.13}$$

And

$$c_1 e^{\left(\sqrt{\frac{Ha^2+Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}}\right)} + c_2 e^{\left(-\sqrt{\frac{Ha^2+Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}}\right)} + \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} = 0 \tag{4.14}$$

Let

$$\lambda = \sqrt{\frac{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}}{\mu_0 e^{\gamma+1+\beta\theta_{\max}}}} \tag{4.15}$$

Equation (4.13) becomes

$$c_1 e^{-\lambda} + c_2 e^{\lambda} + \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} = 0 \tag{4.16}$$

And Equation (4.14) becomes

$$c_1 e^{\lambda} + c_2 e^{-\lambda} + \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} = 0 \tag{4.17}$$

We obtain

$$c_1 e^{-\lambda+\lambda} + c_2 e^{2\lambda} + \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} = 0 \tag{4.18}$$

$$c_1 e^{\lambda-\lambda} + c_2 e^{-2\lambda} + \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} = 0 \tag{4.19}$$

Taking

$$G^* = \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} \tag{4.20}$$

$$\therefore \phi = \frac{e^{2\lambda} G^*(e^{\lambda} - e^{-\lambda})}{e^{2\lambda} - e^{-2\lambda}} - G^* e^{\lambda} e^{\lambda y} - \frac{G^*(e^{\lambda} - e^{-\lambda})}{e^{2\lambda} - e^{-2\lambda}} e^{-\lambda y} + \frac{G\theta_{\max} + F}{Ha^2 + Ae^{\gamma+1+\beta\theta_{\max}}} \tag{4.21}$$

5.0 Results and Discussions

The result presented in figures below demonstrate the effects of maximum temperature $\theta_{\max}$, Hartman number (Ha), body force (F), variable viscosity (μ_0) Grashof number (G) and porosity term (A) respectively on MHD flow of a non-Newtonian fluid for equation (2.14).

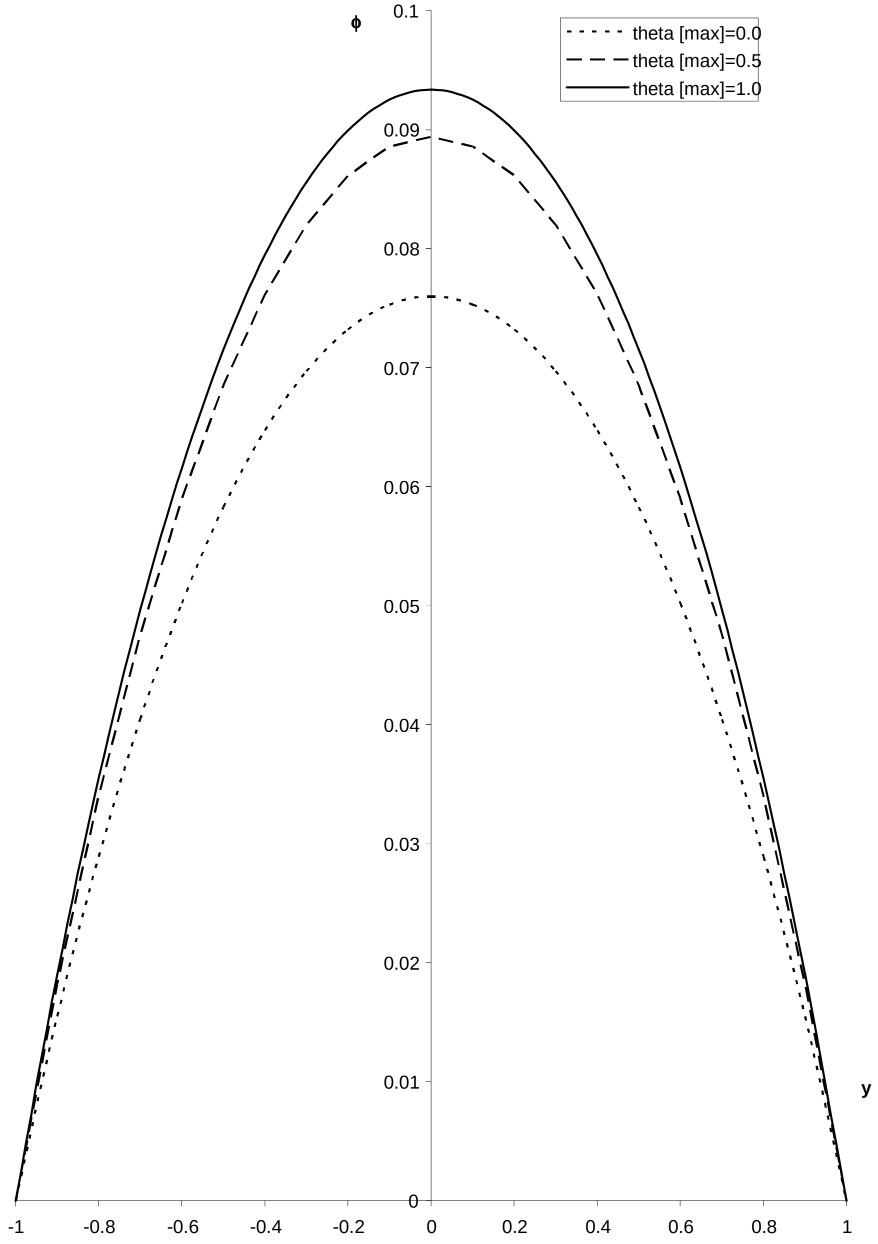

Figure 1: velocity profile for various values of theta [max] when $\gamma=\beta=G=A=F=\mu[0]=Ha=0.5$

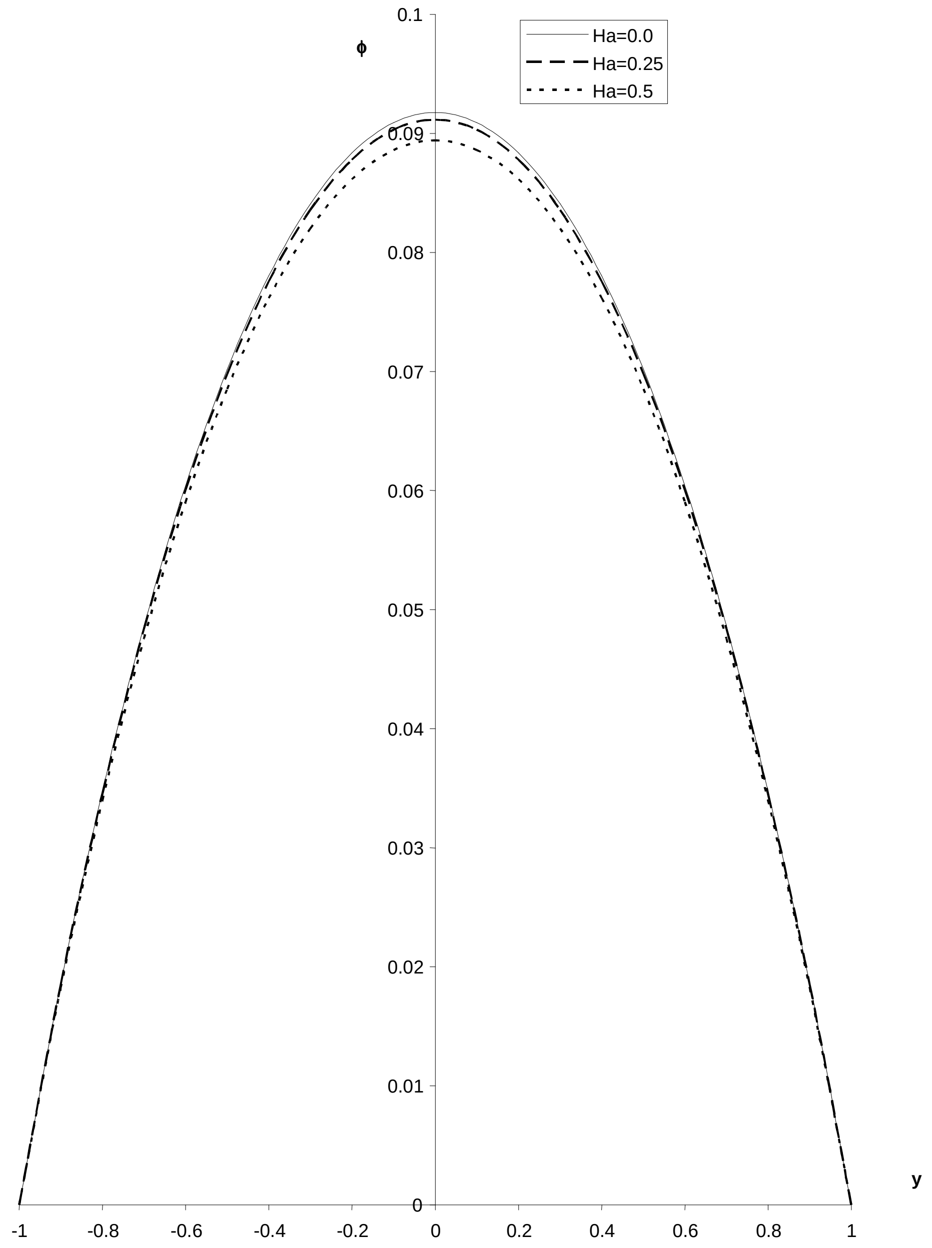

Figure 2: Velocity profile at various values of Ha when theta[max]=A=G=F=mu[0]= γ=β=0.5

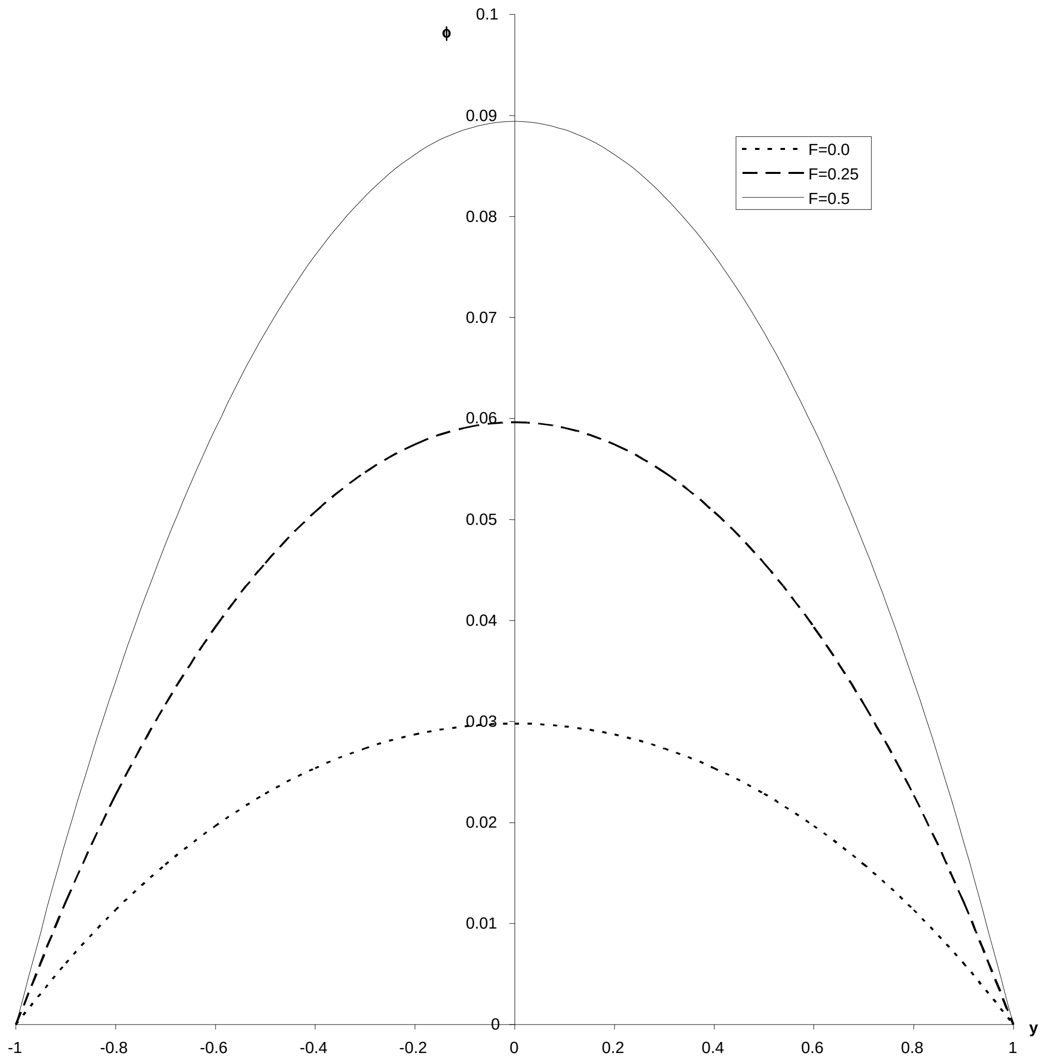

Figure 3: Velocity profile at various values of F when theta[max]=Ha=G=A= γ=β=0.5

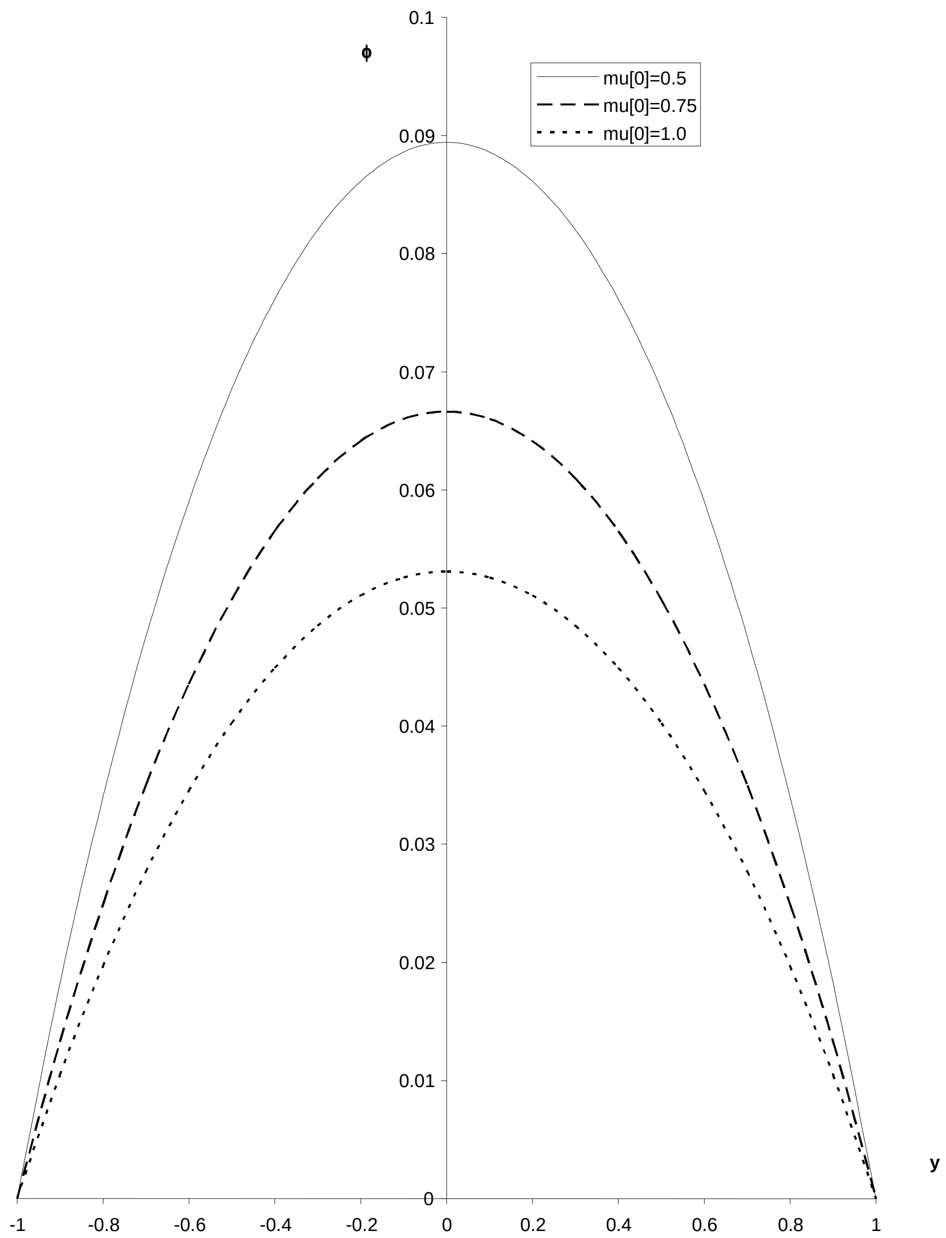

Figure 4: Velocity profile at various values of μ[0] when $\gamma=\beta$=G=A=Ha=theta[max]=F=0.5

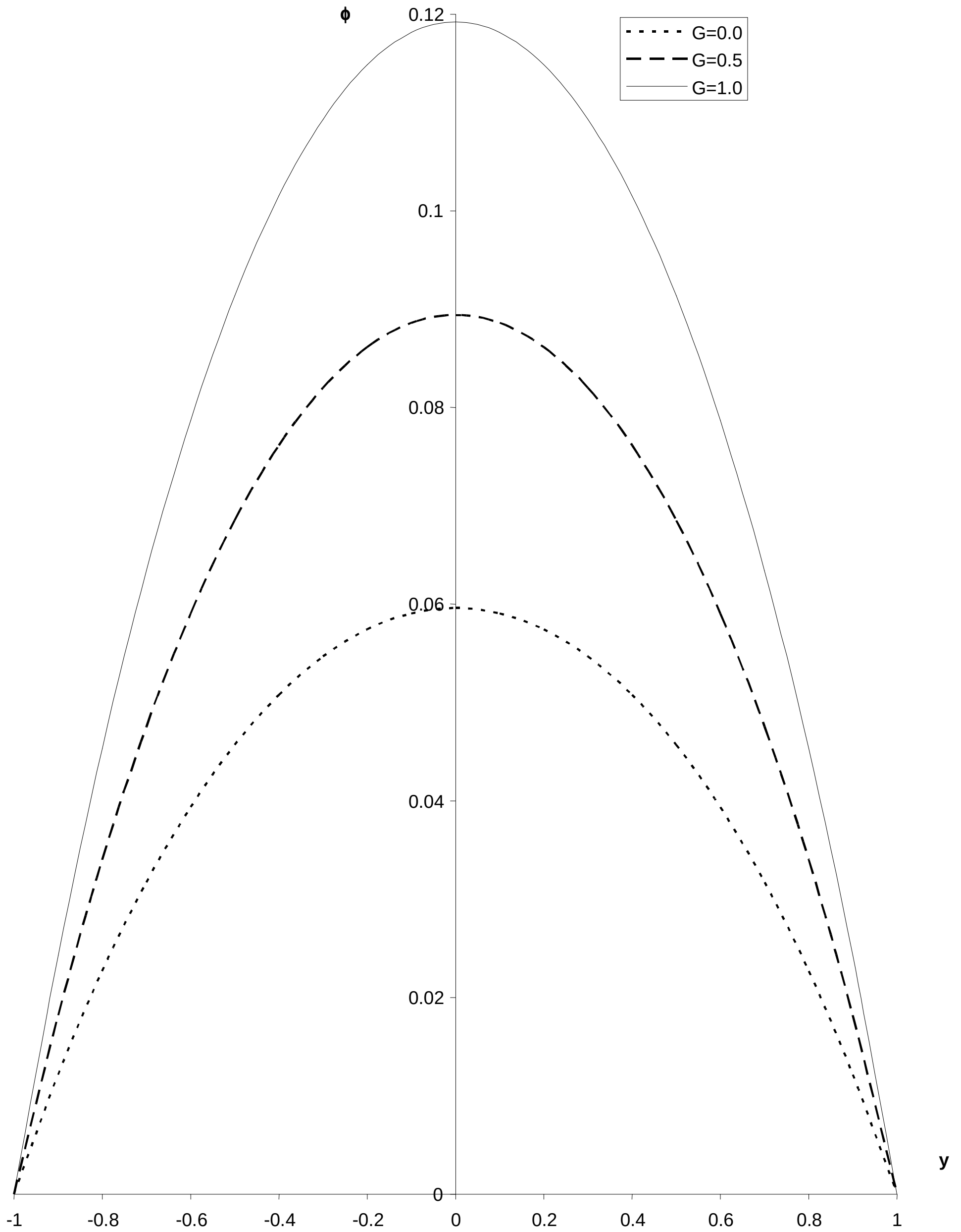

Figure 5: Velocity profile at various values of G when theta[max]= μ[0]=Ha=A=F= γ=β=0.5

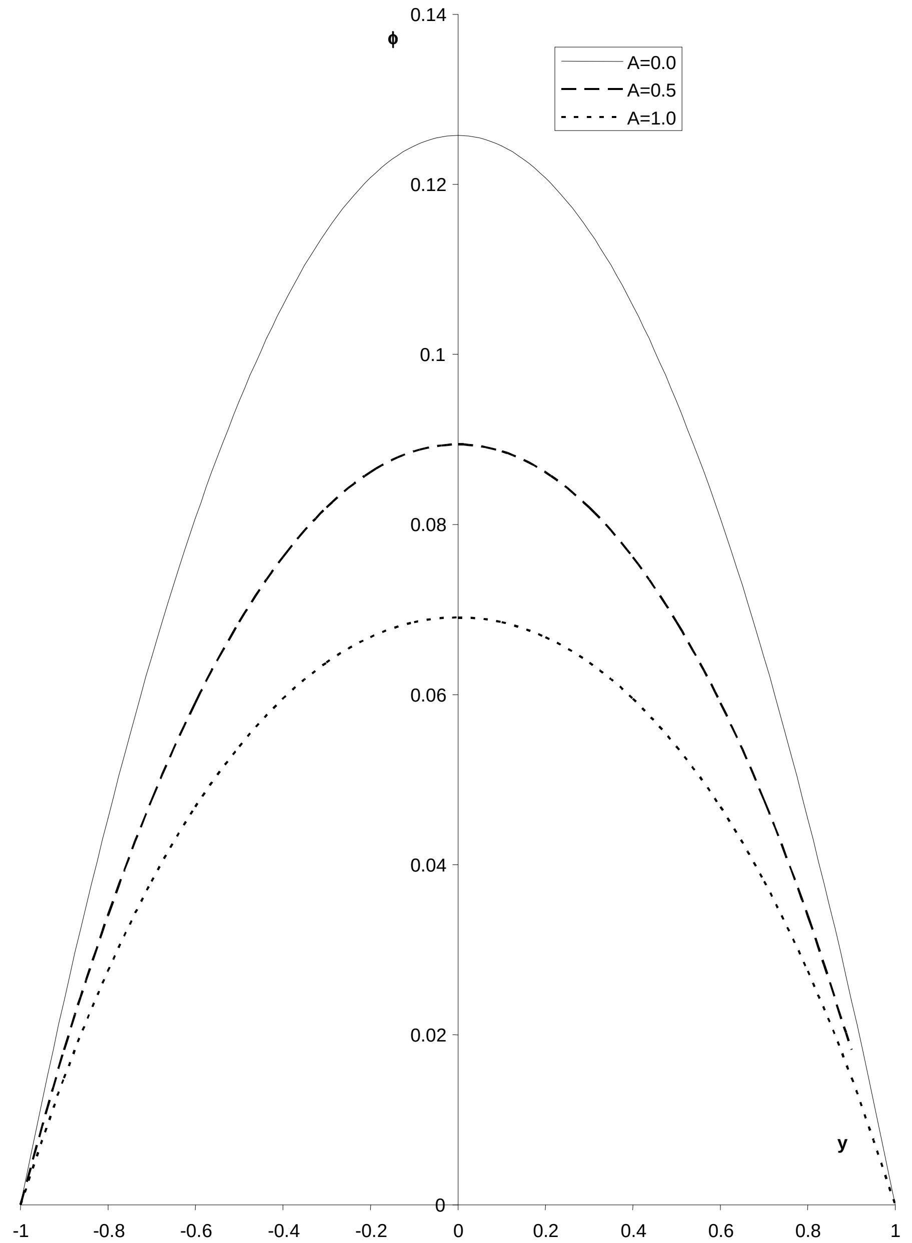

Figure 6: Velocity profile at various values of A when theta[max]= μ[0]=Ha=F=G= γ=β=0.5

Figure 1 show that as $\theta_{\max}$ increases velocity also increase. Figure 2 shows that as Hartmann number (Ha) decreases velocity increases. Figure 3 shows that as body force (F) increases, velocity also increase. Figure 4 shows that as variable viscosity (μ_0) decreases, velocity increase. Figure 5 shows that as Grashof number (G) increases, velocity also increase. Figure 6 shows that as Porosity term (A) decreases, velocity increase.

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