Int. J. Pure Appl. Sci. Technol., 27(1) (2015), pp. 11-16

International Journal of Pure and Applied Sciences and Technology

ISSN 2229 - 6107

Available online at **www.ijopaasat.in**

Research Paper

Generalized Riser Design by Parametric Fuzzy Geometric Programming

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(Received: 1-1-15; Accepted: 25-2-15)

Abstract: *The optimal design of risers for castings has been the subject of numerous investigations and this paper determines the optimal dimensions when Chvorinov's rule for solidification is obeyed by both the riser and casting and solidification time is the only constraint involved .This problem can be formulated to minimize the riser volume subject to the constraint that the riser solidification time is greater than or equal to the casting solidification time. The problem is solved using the Fuzzy geometric programming technique and generalized expressions were obtained for* H_R , D_R *and* V_R *. These expressions were applied to a cylindrical top riser, cylindrical side riser, hemispherical riser and modified hemispherical riser.*

Keywords: Fuzzy geometric programming, Riser design problem, Chvorinov's rule, casting modulus.

Introduction

Geometric programming is an effective method to solve a nonlinear programming problem. It has certain advantages over the other optimization methods. Here, the advantages are that it is usually much simpler to work with the dual than primal. Degree difficulty plays an important role for solving a nonlinear programming problem by geometric programming method. Since late 1960, geometric programming has been known and used in various fields. Duffin and Petersen and Zener (1966) [12] discussed the basic theories of geometric programming. There are many references on application and the method of geometric programming. In the papers like Eckar (1980), Beightler (1979) [2,3], Zener (1971) [4], Jung and Klain (2001) developed single item inventory problems and solved by geometric programming method. Geometric programming has been applied to simple riser problems by R.C.

Creese [10] using Chvorinov's rule [9]. In the last 20 yrs fuzzy geometric programming has received rapid development in the theory and application. In 2002, B.Y. Cao [6] published the first monograph of fuzzy geometric programming as applied optimization series (vol. 76), fuzzy geometric programming by Kluwer academy publishing (the present spinger), the book gives a 2.detailed exposition to theory and application of fuzzy geometric programming. The parametric fuzzy geometric programming can now be applied to a generalized riser design problem. Finally a two degrees difficulty geometric programming problem is solved.

Crisp Model

The model is to be considered such that the riser shape is completely described in terms of its height and diameter. The only restriction which will be considered is that the solidification time of the riser is greater than the solidification time of the casting. The generalized formulation would be

$$\text{Minimize } V_R = f(D_R, H_R)$$
$$\text{Subject to } t_R \geq t_C$$

Where the subscript R applies to the riser and C to the casting and
V_R = riser volume, D_R = riser diameter, H_R = riser height

t_R = solidification time of riser, t_C = solidification time of casting.

If we can assume that the solidification times follow Chvorinov's rule in both the casting and riser such that

$t = B_R(V/SA)_R{}^{n_R}$

and $t = B_C(V/SA)_C{}^{n_C}$

Where B_R = solidification constant for the riser

$(V/SA)_R$= volume-to-surface area ratio for riser

n_R = solidification exponent for riser

B_C = solidification constant for casting

$(V/SA)_C$ = volume-to-surface area ratio for casting

n_C = solidification exponent for casting.

The volume –to-surface area ratio for casting is also called the casting modulus. The values of n_R and n_C vary from 1.5 to 2.5 depending upon the alloy composition and casting shape. The constraint now becomes

$(V/SA)_R \geq (B_C/B_R)^{1/n_R}(V/SA)_C{}^{n_C/n_R}$

Since B_C,B_R , n_C , n_R and $(V/SA)_C$ are constants for a given casting shape and alloy, above equation can be written as $(V/SA)_R \geq Y$

Where Y = riser system modulus.

Let us now use the general relationships for the volume and surface area of the riser wherein
$V_R = AD_R^2H_R + BD_R^3$ and $SA_R = CD_R^2 + KD_RH$

Where A, B, C and K are constants for the various shapes.
So the constraint becomes

$$\frac{AD_R^2H_R+BD_R^3}{CD_R^2+KD_RH_R} \geq Y$$

3. Fuzzy Model

$$\widetilde{Min}\, g_0 = AD_R^2H_R + BD_R^3 \leq V^*$$

Subject to $(V/SA)_R \gtrapprox Y$

Or, $\frac{AD_R^2H_R+BD_R^3}{CD_R^2+KD_RH_R} \gtrapprox Y$

Or, $\frac{AD_RH_R+BD_R^2}{CD_R+KH_R} \gtrapprox Y$

Linear membership functions for the fuzzy objective and constraint goal are

$$\mu_{g_0} = \begin{cases} 1 & if\ g_0 \leq V^* \\ \frac{V^*+P_{V^*}-g_0}{P_{V^*}} & if\ V^* \leq g_0 \leq V^*+P_{V^*} \\ 0 & if\ g_0 \geq V^*+P_{V^*} \end{cases}$$

$$\mu_{g_1} = \begin{cases} 0 & if\ g_1 \leq Y \\ \frac{g_1-Y}{P_Y} & if\ Y \leq g_1 \leq Y+P_Y \\ 1 & if\ g_1 \geq Y+P_Y \end{cases}$$

According to Zimmermann
Max α

Such that $\mu_{g_0} \geq \alpha$
$\mu_{g_1} \geq \alpha$

Which is equivalent to
Min (-α)

Such that $\frac{AD_R^2H_R+BD_R^3}{V(\alpha)} \leq 1$

$$(Y+P_Y\,\alpha)\frac{K}{A}D^{-1} + (Y+P_Y\,\alpha)\frac{C}{A}H^{-1} - \frac{B}{A}D\,H^{-1} \leq 1$$

The dual geometric programming problem is

$$\text{Max } d(w) = \xi_0\left\{\left(\frac{-\alpha}{w_{00}}\right)^{w_{00}}\left(\frac{A}{V(\alpha)w_{01}}\right)^{w_{01}}\left(\frac{B}{V(\alpha)w_{02}}\right)^{w_{02}}\left(\frac{(Y+P_Y\alpha)K}{Aw_{11}}\right)^{w_{11}}\left(\frac{(Y+P_Y\alpha)C}{Aw_{12}}\right)^{w_{12}}\left(\frac{B}{Aw_{13}}\right)^{-w_{13}}(w_{11}+w_{12}-w_{13})^{w_{11}+w_{12}-w_{13}}\right\}^{\xi_0}$$

Such that $w_{00} = 1$

$$w_{01}+w_{02} = w_{00} = 1$$

$2w_{01} + 3w_{02} - w_{11} - w_{13} = 0$

$w_{01} - w_{12} + w_{13} = 0$

Primal-dual variable relations are

$AD_R^2H_R = w_{01}\, d(w)$

$BD_R^3 = w_{02} d(w)$

$$(Y+P_Y\,\alpha)\,\frac{K}{A} D^{-1} = \frac{w_{11}}{w_{11}+w_{12}-w_{13}}$$

$$(Y+P_Y\,\alpha)\,\frac{C}{A} H^{-1} = \frac{w_{12}}{w_{11}+w_{12}-w_{13}}$$

$$\frac{B}{A}\,D\,H^{-1} = \frac{w_{13}}{w_{11}+w_{12}-w_{13}}$$

$W_{00} = 1$, $w_{01} = \frac{2AC-3KB}{2(AC-KB)}$, $w_{02} = \frac{KB}{2(AC-KB)}$, $w_{11} = \frac{2AC-3KB}{AC-KB}$,

$W_{12} = \frac{AC}{AC-KB}$, $w_{13} = \frac{3KB}{2(AC-KB)}$.

$H_R = \frac{2AC-3KB}{A}.\frac{3(Y+P_Y\alpha)}{2A}$, $D_R = \frac{3K}{2A}(Y+P_Y\alpha)$, $V_R = \left(\frac{3(Y+P_Y\alpha)}{2A}\right)^3 .\, 2\,K^2.(CA-BK)$

Also

$$(-\alpha) = d(w) = (-\alpha).-\left(\frac{A}{V(\alpha)\frac{2AC-3KB}{2(AC-KB)}}\right)^{\frac{2AC-3KB}{2(AC-KB)}}\left(\frac{B}{V(\alpha)\frac{KB}{2(AC-KB)}}\right)^{\frac{KB}{2(AC-KB)}}\left(\frac{K(Y+P_Y\alpha)}{A.\frac{2AC-3KB}{(AC-KB)}}\right)^{\frac{2AC-3KB}{(AC-KB)}}$$

$$\left(\frac{C(Y+P_Y\alpha)}{A.\frac{AC}{(AC-KB)}}\right)^{\frac{AC}{(AC-KB)}}\left(\frac{B}{A.\frac{3KB}{2(AC-KB)}}\right)^{\frac{-3KB}{2(AC-KB)}}\left(\frac{3(2AC-3KB)}{2(AC-KB)}\right)^{\frac{3(2AC-3KB)}{2(AC-KB)}}$$

This is a non-linear equation in α.

4. Illustrative Example

$Y = 1$, $Y_\alpha = 0.2$, $V^* = 170$, $P_V{}^* = 10$.

Table: Optimal Riser Design values for several riser Designs

Type of Riser	Riser Design parameters A B C K	Parameter α	Riser Diameter D_R	Riser Height H_R	Riser Volume V_R
Cylindrical Side	π/4 0 π/2 π	0.09	6.11	6.108	179.04

Cylindrical Top	π/4 0 π/4 π	0.45	6.54	3.27	109.89
Hemispherical	π/4 π/12 3π/4 π	0.48	6.57	3.28	185.68
Modified Hemispherical	π/4 7π/162 7π/12 π	0.06	6.07	3.93	144.40

5. Results

The interpretation of these results in the above table leads to some interesting findings. When comparing the three side risers, it is observed that the modified hemispherical bottom has less volume, and thus a greater yield than the other designs. The difference between the hemispherical bottom riser and modified hemispherical bottom is only 1-2 % whereas the hemispherical bottom required 16-17 % less metal than the standard cylindrical side riser. The cylindrical top riser uses 40 % less metal than the cylindrical side riser.

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