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Research Paper

Reverse Derivations on Prime Gamma Near Rings

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Abstract: *The main purpose of this paper is to study and investigate some results concerning reverse derivations on prime Γ-near rings .If M be a prime Γ-near ring with a non zero reverse derivation D, and U be an invariant subset of M then, M is commutative.*

Keywords: Prime Γ-near ring, an invariant set, reverse derivation, commutativity.

1. Introduction:

Bresar and Vukman [1] have introduced the notion of a reverse derivation as an additive mapping d from a ring R into itself satisfying $d(xy) = d(y)x + yd(x)$, for all $x,y \in R$. Samman and Alyamani [3] studied the reverse derivations on semi prime rings .In this paper some results concerning to reverse and right reverse derivations on a non zero invariant set of a prime Γ-near ring are presented.

2. Preliminaries

A Γ-near-ring M is a triple $(M, +, \Gamma)$ where

(i) $(M, +)$ is a not necessarily abelian group,

(ii) Γ is a non-empty set of binary operations on M such that for each $\alpha \in \Gamma$, $(M, +, \alpha)$ is a near-ring.

(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. A subset U of a Γ-near-ring M is said to be left (resp. right) invariant if $x\alpha a \in U$ (resp. $a\alpha x \in U$), for all $a \in U$, $\alpha \in \Gamma$ and $x \in M$. If U is both left and right invariant, we say that U is invariant. Throughout, M will represent a prime Γ-near ring with center $Z(M)$ defined as $Z(M)=\{x \in M: x\alpha y = y\alpha x$, for all $y \in M$ and $\alpha \in \Gamma\}$. We write $[x, y]_\alpha$ for $x\alpha y - y\alpha x$. Recall that a Γ-near ring M is called prime if for $x, y \in M$, $x\Gamma M\Gamma y = \{0\}$ implies $x = 0$ or $y = 0$. An additive mapping D from M into itself is called a derivation if $D(x\alpha y) = D(x)\alpha y + x\alpha D(y)$ for all $x, y\in M$, $\alpha\in\Gamma$, and is called a reverse derivation if $D(x\alpha y) = D(y)\alpha x + y\alpha D(x)$, for all $x, y\in M$, $\alpha\in\Gamma$.

Throughout the present paper we consider M satisfying the assumption (*)…. $x\alpha y\beta z = x\beta y\alpha z$, for all x, $y \in M$ and $\alpha, \beta\in \Gamma$. According to this assumption, we shall make extensive use of the following basic commutator identities:

$[x\beta y, z]_\alpha = x\beta[y, z]_\alpha + [x, z]_\alpha\beta y$,
$[x, y\beta z]_\alpha = y\beta\, [x, z]_\alpha + [x, y]_\alpha\, \beta z$, for all $x, y, z \in M$ and $\alpha, \beta\in \Gamma$.

3. Main Results

In order to prove our results, we need the following lemmas:

Lemma 3.1 [2]: *Let M be a prime Γ-near-ring and U (≠ {0}) be a right (resp. left) invariant subset of M. If x is an element of M such that UΓx = {0} (resp. xΓU = {0}), then x = 0.*

Lemma 3.2 [2]: *Let M be a prime Γ-near-ring and U(≠ {0}) be an invariant subset of M, then for any x, y ∈M ,xΓUΓy = {0} implies x = 0 or y = 0.*

Lemma 3.3: *Let M be a prime Γ-near ring, U(≠ {0})be an invariant subset of M ,and D be a non zero reverse derivation of M, then*

(i) For any $y\in M$, if $D(U)\Gamma y = \{0\}$ then $y = 0$.
(ii) For any $x\in M$, if $x\Gamma D(U) = \{0\}$ then $x = 0$.

Proof:

(i) Assume that $D(U)\Gamma y= \{0\}$, for $y\in M$.

Let $u\in U$ and $\alpha\in\Gamma$, then we have

$D(u)\alpha y = 0.$ ……… (1)

Replacing u by $u\beta x$ in equ. (1) and using (1), we get,

$D(x)\beta\, u\, \alpha y = 0.$ for all $u\in U$, $x,y\in M$ and $\alpha,\beta\in\Gamma$. ……… (2)

By using lemma 3.2 in equ. (2), and since $D\neq 0$, then we have $y = 0$.
(ii) Similarly of proof (i).

Theorem 3.4: *Let M be a prime Γ-near ring, U(≠ {0})be an invariant subset of M ,and D be a non zero reverse derivation of M. If D is commuting on U, then M is commutative.*

Proof:

Let $[D(u),u]_\alpha = 0$, for all $u\in U$ and $\alpha\in\Gamma$. ……… (1)

By linearizing the equ. (1), which gives

$[v, u]_\alpha\, \beta D(u) = 0$, for all $u, v\in U$ and $\alpha,\beta\in\Gamma$. ……… (2)

For $y\in M$, we replace v by $y\lambda v$ in equ. (2) and using (2), we get,

$[y, u]_\alpha \lambda v\ \beta D(u) = 0$, for all $u, v \in U$, $y \in M$ and $\alpha,\beta,\lambda \in \Gamma$. ……… (3)

Now, we taking w instead of $v\beta D(u)$ in equ. (3), we get

$[y, u]_\alpha \lambda w = 0$, for all $u, w \in U$, $y \in M$ and $\alpha,\lambda \in \Gamma$. ……… (4)

Then by using lemma 1, we have

$[y, u]_\alpha = 0$, for all $u \in U$, $y \in M$ and $\alpha \in \Gamma$. ……… (5)

For $x \in M$, we replace u by $x\delta u$ in equ. (5) and using (5), we get,

$[y, x]_\alpha \delta u = 0$, for all $u \in U$, $x,y \in M$ and $\alpha,\delta \in \Gamma$.

Then by using lemma 3.1, we have

$[y, x]_\alpha = 0$, for all $x,y \in M$ and $\alpha \in \Gamma$. Hence M is commutative.

Theorem 3.5: *Let M be a prime Γ-near ring, U(≠{0})be an invariant subset of M ,and D be a non zero reverse derivation of M. If* $[D(v),D(u)]_\alpha = 0$ *,for all* $u,v \in U$ *and* $\alpha \in \Gamma$*, then M is commutative.*

Proof:

Given that $[D(v), D(u)]_\alpha = 0$, for all $u,v \in U$ and $\alpha \in \Gamma$, ……… (1)

Replacing v by $v\beta$u in equ. (1) and using (1), we get,

$D(u)\beta[v,D(u)]_\alpha + [u,D(u)]_\alpha \beta D(v) = 0$, for all $u,v \in U$ and $\alpha,\beta \in \Gamma$, ……… (2)

Replacing v by $z\lambda v$, where $z \in Z(M)$ in equ. (2) and using (2), we get,

$[u, D(u)]_\alpha \beta\ v\ \lambda D(z) = 0$, for all $u, v \in U$, $z \in Z$ and $\alpha,\beta,\lambda \in \Gamma$. ……… (3)

By using lemma 3.2, in equ. (3), we have

Either $[u, D(u)]_\alpha = 0$ or $D(z) = 0$ and since $D(z) \neq 0$, then we get

$[u, D(u)]_\alpha = 0$, for all $u \in U$ and $\alpha \in \Gamma$.

By using the similar procedure as in Theorem 3.4, we get, M is commutative.

Theorem 3.6: *Let M be a prime Γ-near ring, U(≠{0})be an invariant subset of M ,and D be a non zero right reverse derivation of M. If* $[D(v),D(u)]_\alpha = [v,u]_\alpha$ *, for all* $u,v \in U$ *and* $\alpha \in \Gamma$*, then M is commutative.*

Proof:

Let $[D(v),D(u)]_\alpha = [v,u]_\alpha$, for all $u,v \in U$ and $\alpha \in \Gamma$, ……… (1)

Replacing v by $v\beta u$ in equ. (1), we get,

$[D(u)\beta v,D(u)]_\alpha + [D(v)\beta u,D(u)]_\alpha = [v,u]_\alpha \beta u$, for all $u,v \in U$ and $\alpha,\beta \in \Gamma$, ……… (2)

By using the definition of right reverse derivation and using equ. (1) in equ. (2),we get

$D(u)\beta[v,D(u)]_{\alpha} + D(v)\beta\,[u,D(u)]_{\alpha} = 0$, for all $u,v\in U$ and $\alpha,\beta\in\Gamma$

By using the similar procedure as in equation (2) of theorem 3.5, the proof is completed. Hence *M* is commutative.

Theorem 3.7: *Let M be a prime Γ-near ring, U(≠{0})be an invariant subset of M ,and D be anon zero reverse derivation of M .If* $D([u,v]_{\alpha})\in Z(M)$*, for all* $u,v\in U$ *and* $\alpha\in\Gamma$*.Then, M is commutative.*

Proof:

Assume that $D([u,v]_{\alpha})\in Z(M)$, for all $u,v\in U$ and $\alpha\in\Gamma$, ……… (1)

Hence for all $u,v\in U$, $m\in M$ and $\alpha,\beta\in\Gamma$,

$[D([u,v]_{\alpha}), m]_{\beta} = 0$, ……… (2)

Replacing $[u,v]_{\alpha}$ by $[u,v]_{\alpha}\delta z$ in equ. (2), we get

$[D(z)\delta\,[u,v]_{\alpha}, m]_{\beta} + [z\delta D\,([u,v]_{\alpha}), m]_{\beta} = 0$, for all $u,v\in U$, $z,m\in M$ and $\alpha,\beta,\delta\in\Gamma$ ……… (3)

By using the commutator identities and using equ. (2) in equ. (3), we get

$[D(z)\beta[u,v]_{\alpha}, m]_{\lambda} = 0$, for all $u,v\in U$, and $\alpha,\beta,\lambda\in\Gamma$ ……… (4)

Replacing $D(z)\beta[u,v]_{\alpha}$ by $[u',v']_{\alpha}$ in equ. (4), we get

$[[u',v']_{\alpha}, m]_{\lambda} = 0$, for all $u',v'\in U$, $m\in M$ and $\alpha,\lambda\in\Gamma$. ...…… (5)

Replacing $[u',v']_{\alpha}$ by $n\gamma[u',v']_{\alpha}$, $n\in M$ in equ. (5) and using (5), we get

$[n, m]_{\lambda}\gamma\,[u',v']_{\alpha} = 0$, for all $u',v'\in U$, $n,m\in M$ and $\alpha,\lambda,\gamma\in\Gamma$. ………. (6)

By using lemma 3.1 in equ. (6), we get

$[n, m]_{\lambda} = 0$, for all $n,m\in M$ and $\lambda\in\Gamma$. Hence *M* is commutative.

Theorem 3.8: *Let M be a prime Γ-near ring, U(≠{0})be an invariant subset of M ,and D be a non zero reverse derivation of M .If* $[u,D(v)]_{\alpha}\in Z(M)$*, for all* $u,v\in U$ *and* $\alpha\in\Gamma$*. Then, M is commutative.*

Proof:

Assume that $[u,D(v)]_{\alpha}\in Z(M)$, for all $u,v\in U$ and $\alpha\in\Gamma$, ……… (1)

Hence for all $u,v\in U$, $m\in M$ and $\alpha,\beta\in\Gamma$, $[[u,D(v)]_{\alpha}, m]_{\beta} = 0$ ..…….. (2)

Replacing u by $u\delta D(v)$ in equ. (2) and using equ. (2), we get

$[u,D(v)]_{\alpha}\,\delta\,[D(v), m]_{\beta} = 0$, for all $u,v\in U$, $m\in M$ and $\alpha,\beta,\delta\in\Gamma$, ……….(3)

By using equation (1) in equ. (3),we get

$r\lambda[u,D(v)]_\alpha\ \delta\ [D(v), m]_\beta = 0$, for all $u,v\in U, m,r\in M$ and $\alpha,\beta,\delta,\lambda \in\Gamma$, ………. (4)

Replacing $[u,D(v)]_\alpha$ by w in equ. (4), we get

$r\lambda w\delta\ [D(v), m]_\beta = 0$, for all $v,w\in U$, $m,r\in M$ and $\beta,\delta,\lambda \in\Gamma$, ………. (5)

By using lemma 3.2 in equ. (5),we get

$[D(v), m]_\beta = 0$, for all $v\in U$, $m\in M$ and $\beta\in\Gamma$, ………. (6)

Replacing $D(v)$ by $n\gamma D(v)$ in equ. (6) and using (6), we get

$[n, m]_\beta\ \gamma\ D(v) = 0$, for all $v\in U$, $n, m\in M$ and $\beta,\gamma \in\Gamma$, ………. (7)

By using lemma 3.3 in equ. (7), we get

$[n, m]_\beta = 0$, for all $n,m\in M$ and $\beta\in\Gamma$. Hence M is commutative.

Theorem 3.9: *Let M be a prime Γ-near ring, U(≠{0})be an invariant subset of M ,and D be a non zero reverse derivation of M .If one of the properties satisfying:*

(i) $D([u,v]_\alpha) = [u,D(v)]_\alpha$,
(ii) $[u,D(v)]_\alpha = [u,v]_\alpha$.

Then, M is commutative.

Proof:

(i) Given that $D([u,v]_\alpha) = [u,D(v)]_\alpha$, for all $u,v\in U$ and $\alpha\in\Gamma$, ……… (1)

Replacing v by $v\beta u$ in equ. (1) and using (1), we get,

$[u,D(u)]_\alpha\ \beta\ v = 0$, for all $u,v\in U$ and $\alpha,\beta\in\Gamma$, ……… (2)

Replacing $D(u)$ by $y\gamma D(u)$, $y\in M$ in equ. (2) and using (2), we get

$[u, y]_\alpha\ \gamma\ D(u)\ \beta\ v = 0$, for all $u,v\in U$, $y\in M$ and $\alpha,\beta,\gamma \in\Gamma$, ………. (3)

Replacing $D(u)\beta v$ by $D(w)$ in equ. (3) and using (3), we get

$[u, y]_\alpha\ \gamma\ D(w) = 0$, for all $u,v\in U$, $y\in M$ and $\alpha,\beta,\gamma \in\Gamma$, ………. (4)

By using lemma 3.3 in equ. (4), we get

$[u, y]_\alpha = 0$, for all $y\in M$, $u\in U$ and $\alpha \in\Gamma$, ………. (5)

Replacing u by $x\delta u$ in equ. (5) and using (5), we get,

$[x,y]_\alpha\ \delta\ v = 0$, for all $v\in U$, $x,y\in M$ and $\alpha,\delta \in\Gamma$. ………. (6)

Then, by using lemma.3.1 in equ. (6), we get

$[x,y]_\alpha = 0$, for all $x,y\in M$ and $\alpha\in\Gamma$.

Hence M is commutative.

(ii) By using the similar procedure as in proof (i), we get, M is commutative.

4. Conclusions:

In this paper the notion of reverse derivations on prime Γ-near ring M can be used to study the commutativity conditions of M, when U is a non zero invariant subset of M.

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