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Research Paper

Topological Property of θ-Semi-Open Sets

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Abstract: *The concept of θ-semi-open sets in topological spaces was first introduced in 1986 by T. Noiri. Now, in the present paper, we introduce and study topological properties of θ-semi-neighborhood, θ-semi-interior, θs-derived, θ-semi-boundary, θ-semi-exterior and θs-border of a set via the concept of θ-semi-open sets. Also, we find several related among them.*

Keywords: θ-semi-open sets, θs-derived set, θ-semi-boundary, θ-semi-exterior.

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1. Introduction

The symbols X and Y represent topological spaces with no separation axioms assumed unless explicitly stated. Let S be a subset of X, the interior and closure of S are denoted by Int(S) and Cl(S), respectively. A subset S of X is said to be semi-open [5] if and only if $S \subset Cl(Int((S)))$. A subset S of X is said to be θ-open [8] (resp. θ-semi-open [7]) set if for each $x \in S$, there exists an open (resp. semi-open) set G in X such that $x \in G \subset Cl(G) \subset S$. The complement of each semi-open (resp. θ-semi-open) sets is called semi-closed (resp. θ-semi-closed). A point x is said to be in the θ-semi-closure of a set S [3], denoted by $sCl_\theta(S)$, if $S \cap Cl(G) \neq \phi$ for each $G \in SO(X)$ containing x. If $S = sCl_\theta(S)$, then S is called θ-semi-closed. For each $G \in SO(X)$, Cl(G) is θ-semi-open. Therefore, $x \in sCl_\theta(S)$ if and only if $S \cap E \neq \phi$ for each θ-semi-open set E containing x. Let A be a subset of a space X, a point $x \in X$ is said to be θ-limit point of A [2] if for any θ-open set U containing x, $U \cap (A \setminus \{x\}) \neq \phi$. The set of all θ-limit points of A is called θ-derived set of A and is denoted by θd(A).

2. θ-Semi-Open Sets

In this section, we study topological properties of θ-semi-neighborhood, θ-semi-interior, θs-derived, θ-semi-boundary, θ-semi-exterior and θs-border of a set via the concept of θ-semi-open sets.

We start this section with the following definition.

Definition 2.1: Let X be a topological space, let $x \in X$ and $M \subset X$. Then, M is called θ-semi-neighborhood of x if there exists a θ-semi-open set A of x such that $x \in A \subset M$.

Theorem 2.2: Let X be a topological space and $x \in X$.

(i) Let A, B be two subsets of X such that $A \subset B$. If A is a θ-semi-neighborhood of x, then B is a θ-semi-neighborhood of x.
(ii) For each $i \in I$, if A_i is a θ-semi-neighborhood of x, then $\cup \{A_i: i \in I\}$ is θ-semi-neighborhood of x.
(iii) A is θ-semi-open if and only if it is a θ-semi-neighborhood of each of it‘s points.

Proof:

(i) It is obvious.

(ii) If A_i is θ-semi-neighborhood of x for each $i \in I$, there exists $U_i \in \theta SO(X)$ such that $x \in U_i \subset A_i$. Let $V = \bigcup_{i \in I} U_i$, then by [4, Lemma 2.2], it follows that $V \in \theta SO(X)$. Hence, $x \in V \subset \cup \{A_i: i \in I\}$. Therefore, $\cup \{A_i: i \in I\}$ is θ-semi-neighborhood of x.

(iii) The “only if” part is obvious. To show the “if” part, suppose A is a θ-semi-neighborhood of each of it‘s points. So, for each point $x \in A$, there is a θ-semi-open set A_x such that $x \in A_x \subset A$. Hence $A = \cup \{A_x: x \in I\}$ and it is θ-semi-open set by [4, Lemma 2.2].

Definition 2.3 [7]: A point $x \in X$ is said to be a θ-semi-interior point of A if there exists a semi-open set U containing x such that $U \subset Cl(U) \subset A$. The set of all θ-semi-interior points of A is said to be the θ-semi-interior of A and is denoted by $sInt_\theta(A)$.

Theorem 2.4: Let X be a topological space. Let A and B be two subsets of X. Then,

(1) $sInt_\theta (A \cap B) \subset (sInt_\theta(A) \cap sInt_\theta(B))$.

(2) $\bigcup_{i \in I} sInt_\theta (A_i) \subset sInt_\theta (\bigcup_{i \in I} A_i)$.

(3) $sInt_\theta (A \setminus B) \subset (sInt_\theta (A) \setminus B) \subset (sInt_\theta (A) \setminus sInt_\theta (B))$.

Proof:

(1) It is obvious.

(2) Since $sInt_\theta (A_i) \in \theta SO(X)$, then $\bigcup_{i \in I} sInt_\theta (A_i) \in \theta SO(X)$ [10, Lemma 2.2] and by [1, Remark 1.2.5], $sInt_\theta (\bigcup_{i \in I} sInt_\theta (A_i)) = \bigcup_{i \in I} sInt_\theta (A_i)$. But we know that $sInt_\theta (\bigcup_{i \in I} sInt_\theta (A_i)) \subset sInt_\theta (\bigcup_{i \in I} A_i)$. So, $\bigcup_{i \in I} sInt_\theta (A_i) \subset sInt_\theta (\bigcup_{i \in I} A_i)$.
(3) Let $x \in sInt_\theta (A \setminus B)$. Then, $x \in (A \setminus B)$. So, $x \in A$ and $x \notin B$. On the other hand, since $x \in sInt_\theta (A \setminus B)$, there exists $U \in \theta SO(X)$ such that $x \in U \subset (A \setminus B) \subset A$. Hence, $x \in sInt_\theta (A)$. Therefore, $x \in (sInt_\theta (A) \setminus B)$. Thus, $sInt_\theta (A \setminus B) \subset (sInt_\theta (A) \setminus B)$. Also, $x \notin sInt_\theta (B)$ since $x \notin B$. So, $x \in (sInt_\theta (A) \setminus sInt_\theta (B))$. Thus, $sInt_\theta (A \setminus B) \subset (sInt_\theta (A) \setminus B) \subset (sInt_\theta (A) \setminus sInt_\theta (B))$.

Proposition 2.5: Let X be a topological space and A, B be subsets of X. Then,
$sCl_\theta (A \cap B) \subset (sCl_\theta (A) \cap sCl_\theta (B))$.

Proof:

It is obvious.

Definition 2.6 [1]: Let A be a subset of a space X. A point $x \in X$ is said to be θs-limit point of A if for each θ-semi-open set U containing x, $U \cap (A \setminus \{x\}) \neq \phi$. The set of all θs-limit points of A is called the θs-derived set of A and is denoted by θsd(A).

Theorem 2.7: For subsets A, B of a space X, the following statements hold:

(1) $\theta d(A) \subset \theta sd(A)$.
(2) If $A \subset B$, then $\theta sd(A) \subset \theta sd(B)$.
(3) $\theta sd(A) \cup \theta sd(B) = \theta sd(A \cup B)$ and $\theta sd(A \cap B) \subset (\theta sd(A) \cap \theta sd(B))$.
(4) $(\theta sd(\theta sd(A)) \setminus A) \subset \theta sd(A)$.
(5) $\theta sd(A \cup \theta sd(A)) \subset (A \cup \theta sd(A))$.

Proof:

(1) It suffices to observe that every θ-open set is θ-semi-open.
(2) Obvious.
(3) They are modification of the proof for θd(A), where θ-open sets are replaced by θ-semi-open sets.
(4) If $x \in (\theta sd(\theta sd(A)) \setminus A)$ and U is a θ-semi-open set containing x, then $U \cap (\theta sd(A) \setminus \{x\}) \neq \phi$. Let $y \in (U \cap (\theta sd(A) \setminus \{x\}))$. Then, since $y \in \theta sd(A)$ and $y \in U$, $U \cap (A \setminus \{y\}) \neq \phi$. Let $z \in (U \cap (A \setminus \{y\}))$. Then, $z \neq x$ for $z \in A$ and $x \notin A$. Hence $U \cap (A \setminus \{x\}) \neq \phi$. Therefore, $x \in \theta sd(A)$.
(5) Let $x \in \theta sd(A \cup \theta sd(A))$. If $x \in A$, the result is obvious. So, let $x \in (\theta sd(A \cup \theta sd(A)) \setminus A)$, then for θ-semi-open set U containing x, $U \cap (A \cup \theta sd(A) \setminus \{x\}) \neq \phi$. Thus, $U \cap (A \setminus \{x\}) \neq \phi$ or $U \cap \theta sd(A) \setminus \{x\}) \neq \phi$. Now, it follows similarly from (4) that $U \cap (A \setminus \{x\}) \neq \phi$. Hence $x \in \theta sd(A)$. Therefore, in any case $\theta sd(A \cup \theta sd(A)) \subset (A \cup \theta sd(A))$.

Theorem 2.8: For a subset A of a space X, the following statement is true.
$sInt_\theta (A) = A \setminus \theta sb(X \setminus A)$.

Proof:

If $x \in [A \setminus \theta sb(X \setminus A)]$, then $x \notin \theta sb(X \setminus A)$ and so there exists a θ-semi-open set U containing x such that $U \cap (X \setminus A) = \phi$. Then, $x \in U \subset A$ and hence $x \in sInt_\theta (A)$, that is, $[A \setminus \theta sb(X \setminus A)] \subset sInt_\theta (A)$.

On the other hand, if $x \in sInt_\theta (A)$, then $x \notin \theta sb(X \setminus A)$ since $sInt_\theta (A)$ is θ-semi-open and $[sInt_\theta (A) \cap (X \setminus A)] = \phi$. Hence, $sInt_\theta (A) = A \setminus \theta sb(X \setminus A)$.

Definition 2.9: Let X be a topological space and $A \subset X$. A point $x \in X$ is called semi-boundary [6](resp. θ-semi-boundary) point of A if and only if each semi-open(resp. θ-semi-open) set containing x has a non empty intersection with A and $(X \setminus A)$. The set of all semi-boundary (resp. θ-semi-boundary) points of A is called semi-boundary(resp. θ-semi-boundary) of A and is denoted by sb(A)(resp. θsb(A)).

Proposition 2.10: Let X be a topological space and $A \subset X$. Then, $sb(A) \subset \theta sb(A)$.

Proof:

Let $x \in sb(A)$, then for each semi-open set U containing x we have $U \cap A \neq \phi$ and $U \cap (X \setminus A) \neq \phi$. But $\theta SO(X) \subset SO(X)$. So, for each θ-semi-open set U containing x we have $U \cap A \neq \phi$ and $U \cap (X \setminus A) \neq \phi$. Thus, $x \in \theta sb(A)$ and hence $sb(A) \subset \theta sb(A)$.

Definition 2.11: Let X be a topological space and $A \subset X$. A point $x \in X$ is called semi-exterior [6] (resp. θ-semi-exterior) point of A if and only if there exists a semi-open (resp. θ-semi-open) set U containing x such that $A \cap U = \phi$. The set of all semi-exterior(resp. θ-semi-exterior) points of A is called semi-exterior (resp. θ-semi-exterior) of A and is denoted by sE(A) (resp. θsE(A)).

Proposition 2.12: Let X be a topological space and $A \subset X$. Then, $sE(A) \subset \theta sE(A)$.

Proof:

Let $x \in sE(A)$, there exists a semi-open set U in X containing x such that $U \cap A = \phi$. Since $\theta SO(X) \subset SO(X)$, then U is θ-semi-open in X containing x and $U \cap A = \phi$, thus we have $x \in \theta sE(A)$. Hence $sE(A) \subset \theta sE(A)$.

Theorem 2.13: Let X be a topological space and $A \subset X$. Then,
(1) $X = \theta sb(A) \cup sInt_\theta (A) \cup \theta sE(A)$.
(2) The sets θsb(A), θsE(A) and $sInt_\theta$ (A) are pairwise disjoint.

Proof:

(1) Let $x \in X$ and $x \notin \theta sE(A)$, $x \notin sInt_\theta (A)$ and $x \notin \theta sb(A)$. Since $x \notin \theta sE(A)$, then for each $U \in \theta SO(X)$ containing x, $U \cap A \neq \phi$ … (a)
Since $x \notin sInt_\theta (A)$, then for each $V \in \theta SO(X)$ containing x, $V \subset A$ … (b)

Since $x \notin \theta sb(A)$, there exists $W \in \theta SO(X)$ containing x such that $W \cap A = \phi$ or $W \cap (X \setminus A) = \phi$ … (c)

From (c) we get $W \cap A = \phi$. This contradicts (a). Also, from (c) we obtain $W \cap (X \setminus A) = \phi$, i.e., $W \subset A$. This contradicts (b). Therefore, each point in X belongs to one and only one of the sets θsb(A), θsE(A) and $sInt_\theta$ (A).

(2) To show that $\theta sb(A) \cap sInt_\theta (A) = \phi$, suppose not. Let $x \in (\theta sb(A) \cap sInt_\theta (A))$. Then, $x \in \theta sb(A)$ and $x \in sInt_\theta (A)$. Since $x \in sInt_\theta (A)$, there exists $U \in \theta SO(X)$ containing x and $U \subset A$. Since $x \in \theta sb(A)$, then for each $V \in \theta SO(X)$ containing x we have $V \cap A \neq \phi$ and $V \cap (X \setminus A) \neq \phi$. Hence $V \not\subset A$, which is a contradiction. To show $\theta sb(A) \cap \theta sE(A) = \phi$, suppose not, let $x \in (\theta sb(A) \cap \theta sE(A))$, then $x \in \theta sb(A)$ and $x \in \theta sE(A)$, since $x \in \theta sE(A)$, there exists $U \in \theta SO(X)$ containing x and $U \cap A = \phi$. So, $U \subset (X \setminus A)$, i.e., $U \cap A = \phi$. Hence $x \notin \theta sb(A)$. This is a contradiction. To show that $sInt_\theta (A) \cap \theta sE(A) = \phi$, suppose not, let $x \in (sInt_\theta (A) \cap \theta sE(A))$. Then, $x \in sInt_\theta (A)$ and $x \in \theta sE(A)$. Since $x \in sInt_\theta (A)$, there exists $U \in \theta SO(X)$ containing x and $U \subset A$. Thus, $U \cap A \neq \phi$. This means that $x \notin \theta sE(A)$. This is a contradiction.

Theorem 2.14: If X is a topological space and $A \subset X$, then $\theta sb(A) = \theta sb(X \setminus A)$ and θsb(A) is θ-semi-closed set, for each $A \subset X$.

Proof:

To prove $\theta sb(A) = \theta sb(X \setminus A)$, it is obvious. It remains to prove $\theta sb(A)$ is θ-semi-closed. Now,

$$\begin{aligned} sCl_\theta[\theta sb(A)] &= sCl_\theta[\, sCl_\theta(A) \cap sCl_\theta(X \setminus A)]. \\ &\subset sCl_\theta[\, sCl_\theta(A)] \cap sCl_\theta[\, sCl_\theta(X \setminus A)]. \\ &= sCl_\theta(A) \cap sCl_\theta(X \setminus A) \\ &= \theta sb(A). \end{aligned}$$

Hence, $\theta sb(A)$ is θ-semi-closed.

Definition 2.15 [1]: Let X be a topological space and $B \subset X$. A set B is called θs-clopen if and only if it is both θ-semi-open and θ-semi-closed sets.

Theorem 2.16: Let X be a topological space and $A \subset X$. Then,

(1) A is θ-semi-open in X if and only if A contains non of its θ-semi-boundary points.
(2) A is θ-semi-closed in X if and only if A contains all of its θ-semi-boundary points.

Proof:

(1) Let A be θ-semi-open in X. We shall prove that $A \cap \theta sb(A) = \phi$. Suppose not, there exists a point $x \in (A \cap \theta sb(A))$. Hence $x \in A$ and $x \in \theta sb(A)$. Put U = A, since $x \in \theta sb(A)$, then $U \cap A \neq \phi$ and $U \cap (X \setminus A) \neq \phi$, which is contradiction. Therefore, $A \cap \theta sb(A) = \phi$.
Conversely, let A be a set containing non of its θ-semi-boundary points. We shall prove that A is θ-semi-boundary points and A is θ-semi-open. Let $x \in A$. Then, $x \notin \theta sb(A)$. Since $x \notin \theta sE(A)$, it follows that $x \in sInt_\theta(A)$. This shows that $A \subset sInt_\theta(A)$. Since $sInt_\theta(A) \subset A$, it follws that $A = sInt_\theta(A)$ and hence A is θ-semi-open.
(2) A is θ-semi-closed if and only if $(X \setminus A)$ is θ-semi-open if and only if $\theta sb(X \setminus A) \subset A$ (by Part (1)) if and only if $\theta sb(A) = \theta sb(X \setminus A) \subset A$.

Corollary 2.17: Let A be a subset of a topological space X. Then, $\theta sb(A) = \phi$ if and only if A is θs-clopen.

Proof:

A is θs-clopen if and only if A is both θ-semi-open and θ-semi-closed if and only if $\theta sb(A) \cap A = \phi$ and $\theta sb(A) \subset A$ if and only if $\theta sb(A) = \phi$.

Remark 2.18: Before we give relations among $sCl_\theta(A)$, $\theta sb(A)$, $\theta sE(A)$ and $sInt_\theta(A)$, we consider the following:
Suppose A is a subset of a space X and $x \in \theta sb(A \cup \theta sb(A))$. We can show that $x \in \theta sb(A)$. To do this, we first observe that if U is any θ-semi-open set containing x, then $U \cap (A \cup \theta sb(A)) \neq \phi$ and $U \cap (X \setminus (A \cup \theta sb(A)) \neq \phi$. Then,

$$(U \cap A) \cup (U \cap \theta sb(A)) \neq \phi \quad \ldots \quad (1)$$

and

$$U \cap (X \setminus A) \cap (X \setminus \theta sb(A)) \neq \phi \quad \ldots \quad (2)$$

So, from (1) either $U \cap A \neq \phi$ or $U \cap \theta sb(A) \neq \phi$.
Also, from (2) it follows that $U \cap (X \setminus A) \neq \phi$.

$$\text{Now, } U \cap A \neq \phi \text{ and } U \cap (X \setminus A) \neq \phi \quad \ldots \quad (3)$$

Or $U \cap \theta sb(A) = \phi$ and $U \cap (X \setminus A) \neq \phi$ … (4)
If (3) holds, then $x \in \theta sb(A)$.

If (4) holds and $x \in (U \cap \theta sb(A))$, then $U \in \theta SO(X)$ containing x, so that $U \cap A \neq \phi$ and $U \cap (X \setminus A) \neq \phi$ and hence $x \in \theta sb(A)$. Consequently, if $x \in \theta sb(A \cup \theta sb(A))$, then $x \in (A \cup \theta sb(A))$. So that $A \cup \theta sb(A)$ contains all its θ-semi-boundary points and therefore, it is θ-semi-closed by Theorem 2.16(2).
Knowing this, we can express the θ-semi-closure of a set interms of the θ-semi-boundary of the set.

Theorem 2.19: Let X be a topological space and $A \subset X$. Then,
(i) $sCl_\theta (A) = A \cup \theta sb(A)$.
(ii) $sCl_\theta (A) = sInt_\theta (A) \cup \theta sb(A)$.

Proof:

We shall prove (i) only and (ii) can be proved in the same way. The set $A \cup \theta sb(A)$ is θ-semi-closed by Remark 2.18. It will now be shown that $A \cup \theta sb(A)$ is the smallest θ-semi-closed set containing A. Suppose there exists a θ-semi-closed set F containing A such that F is a proper subset of $A \cup \theta sb(A)$. Then, there is a point $x \in (A \cup \theta sb(A))$ such that $x \notin F$. This means that $x \in A$ or $x \in \theta sb(A)$. Since $A \subset F$, it follows that $x \notin A$ and $x \notin \theta sb(A)$. Since F is θ-semi-closed, then $(X \setminus F)$ is a θ-semi-open set containing x, but no point of $A \subset F$. This implies that $x \in \theta sE(A)$. We have now $x \in \theta sE(A)$ and $x \in \theta sb(A)$ which is contradiction by Theorem 2.13(2). Then, suppose the existence of a θ-semi-closed set F smaller than $A \cup \theta sb(A)$ and containing A is false. So, $A \cup \theta sb(A)$ is the smallest θ-semi-closed set containing A, thus $sCl_\theta (A) = A \cup \theta sb(A)$.

Theorem 2.20: Let X be a topological space and $A \subset X$, then
$\theta sb(A) = sCl_\theta (A) \cap sCl_\theta (X \setminus A)$.

Proof:

$x \in \theta sb(A)$ if and only if every θ-semi-open set contains x has a non empty intersection with both A and $(X \setminus A)$ if and only if $x \in sCl_\theta (A)$ and $x \in sCl_\theta (X \setminus A)$ if and only if $x \in (sCl_\theta (A) \cap sCl_\theta (X \setminus A))$.

Theorem 2.21: Let X be a topological space and $A \subset X$, then $\theta sb(A) = (sCl_\theta (A) \setminus sInt_\theta (A))$.

Proof:

To show that $\theta sb(A) \subset (sCl_\theta (A) \setminus sInt_\theta (A))$, suppose not, there exists $x \in \theta sb(A)$ such that $x \notin (sCl_\theta (A) \setminus sInt_\theta (A))$, hence $x \in sCl_\theta (A)$ and $x \in sInt_\theta (A)$. Since $x \in sInt_\theta (A)$, there exists $U \in \theta SO(X)$ containing x and $U \subset A$. Then, $U \cap (X \setminus A) = \phi$, this is contradiction because $x \in \theta sb(A)$. To show that $(sCl_\theta(A) \setminus sInt_\theta(A)) \subset \theta sb(A)$, let $x \in (sCl_\theta (A) \setminus sInt_\theta (A))$. Then, $x \in sCl_\theta (A)$ and $x \in sInt_\theta (A)$. Hence $x \in A$ or $x \in \theta sb(A)$. If $x \in A$ and $x \notin sInt_\theta (A)$, then for each $U \in \theta SO(X)$ such that $x \in U$ we have $U \cap A \neq \phi$ and $U \cap (X \setminus A) \neq \phi$. So, $x \in \theta sb(A)$. Now, suppose $x \in \theta sd(A)$ and $x \notin sInt_\theta (A)$. Then, $x \in \theta sd(A)$ implies that $U \cap A \neq \phi$. Also, $x \notin sInt_\theta (A)$ implies that $U \cap (X \setminus A) \neq \phi$. Hence $x \in \theta sb(A)$. Therefore, $(sCl_\theta (A) \setminus sInt_\theta (A)) \subset \theta sb(A)$. Thus, $\theta sb(A) = (sCl_\theta (A) \setminus sInt_\theta (A))$.

Theorem 2.22: Let X be a topological space and $A \subset X$, then

$$sInt_\theta (A) = (A \setminus \theta sb(A)).$$

Proof:

To show that $sInt_\theta(A) \subset (A \setminus \theta sb(A))$, let $x \in sInt_\theta(A)$. Then, there exists $U \in \theta SO(X)$ containing x and $U \subset A$. Then, $U \cap (X \setminus A) = \phi$ and hence $x \notin \theta sb(A)$. Since $x \in A$ we have $x \in (A \setminus \theta sb(A))$. Therefore, $sInt_\theta(A) \subset (A \setminus \theta sb(A))$. To show that $(A \setminus \theta sb(A)) \subset sInt_\theta(A)$, let $x \in (A \setminus \theta sb(A))$. Then, $x \in A$ and $x \notin \theta sb(A)$. Now, $x \notin \theta sb(A)$ means there exists $U \in \theta SO(X)$ containing x and hence $U \cap (X \setminus A) = \phi$, this means that $U \subset A$ and then $x \in U \subset A$. This implies that $x \in sInt_\theta(A)$. Thus, $(A \setminus \theta sb(A)) \subset sInt_\theta(A)$. Therefore, $sInt_\theta(A) = (A \setminus \theta sb(A))$.
The following result gives a connection between the θ-semi-boundary of a given set and the θ-semi-boundary of its θ-semi-interior.

Theorem 2.23: Let X be a topological space and $A \subset X$. Then,

$$\theta sb(sInt_\theta(A)) \subset \theta sb(A).$$

Proof:

Let $x \in \theta sb(sInt_\theta(A))$. Then, for each $U \in \theta SO(X)$ such that $x \in U$, it follows that $U \cap sInt_\theta(A) \neq \phi$ and $U \cap (X \setminus sInt_\theta(A)) \neq \phi$. If $x \in \theta sb(A)$, there exists $U \in \theta SO(X)$ such that $x \in U$ and either $U \cap A = \phi$ or $U \cap (X \setminus A) = \phi$. Now, $U \cap A \neq \phi$ because $U \cap sInt_\theta(A) \neq \phi$ and $sInt_\theta(A) \subset A$. So, $U \cap (X \setminus A) = \phi$ and hence $U \subset A$. Then, $x \in U \subset A$ and consequently $x \in sInt_\theta(A)$, which is contradiction because $x \in (X \setminus sInt_\theta(A))$. Thus, $x \in \theta sb(A)$ and hence $\theta sb(sInt_\theta(A)) \subset \theta sb(A)$.

Theorem 2.24: Let X be a topological space and A, B be two subsets of X such that $A \subset B$. Then, $\theta sE(B) \subset \theta sE(A)$.

Proof:

Let $x \in \theta sE(B)$, there exists $U \in \theta SO(X)$ such that $x \in U$ and $U \cap B = \phi$, hence $U \cap A = \phi$. Therefore, $x \in \theta sE(A)$. Thus, $\theta sE(B) \subset \theta sE(A)$.

Theorem 2.25: Let X be a topological space and A, B be two subsets of X, then $\theta sE(A \cup B) \subset (\theta sE(A) \cap \theta sE(B))$.

Proof:

Since $A \subset (A \cup B)$, then $\theta sE(A \cup B) \subset \theta sE(A)$ by Theorem 2.24. Similarly, $\theta sE(A \cup B) \subset \theta sE(B)$. Thus, $\theta sE(A \cup B) \subset (\theta sE(A) \cap \theta sE(B))$.

Theorem 2.26: For a subset A of a space X, the following statements hold:

(1) $\theta sE(A)$ is θ-semi-oepn.
(2) $\theta sE(A) = sInt_\theta(X \setminus A) = X \setminus sCl_\theta(A)$.
(3) $\theta sE(\theta sE(A)) = sInt_\theta(sCl_\theta(A))$.
(4) $\theta sE(A) = \theta sE(X \setminus \theta sE(A))$.
(5) $sInt_\theta(A) \subset \theta sE(\theta sE(A))$.

Proof:

(3) $\theta sE(\theta sE(A)) = \theta sE(X \setminus sCl_\theta(A))$.
$= sInt_\theta(X \setminus (X \setminus sCl_\theta(A))$.
$= sInt_\theta(sCl_\theta(A))$.

(4) $\theta sE(X \setminus \theta sE(A)) = \theta sE(X \setminus sInt_\theta(X \setminus A))$.

$= sInt_\theta (X \setminus (X \setminus sInt_\theta (X \setminus A))$.
$= sInt_\theta (sInt_\theta (X \setminus A))$.
$= sInt_\theta (X \setminus A)$.
$= \theta sE(A)$.

(5) $sInt_\theta (A) \subset sInt_\theta (sCl_\theta (A))$.
$= sInt_\theta (X \setminus sInt_\theta (X \setminus A))$.
$= sInt_\theta (X \setminus \theta sE(A))$.
$= \theta sE(\theta sE(A))$.

Definition 2. 27: $\theta sBd(A) = A \setminus sInt_\theta (A)$ is said to be the θs-border of A.

Theorem 2.28: For a subset A of a space X, the following statements hold:

(1) $A = sInt_\theta (A) \cup \theta sBd(A)$.
(2) $sInt_\theta (A) \cap \theta sBd(A) = \phi$.
(3) A is a θ-semi-oepn set if and only if $\theta sBd(A) = \phi$.
(4) $\theta sBd(sInt_\theta (A)) = \phi$.
(5) $sInt_\theta (\theta sBd(A)) = \phi$.
(6) $\theta sBd(\theta sBd(A) = \theta sBd(A)$.
(7) $\theta sBd(A) = A \cap (sCl_\theta (X \setminus A))$.
(8) $\theta sBd(A) = \theta sBd(X \setminus A)$.

Proof:

(5) If $x \in sInt_\theta (\theta sBd(A))$, then $x \in \theta sBd(A)$. On the other hand, since $\theta sBd(A) \subset A$, $x \in sInt_\theta (\theta sBd(A)) \subset sInt_\theta (A)$. Hence, $x \in sInt_\theta (A) \cap \theta sBd(A)$, which contradicts (2). Thus, $sInt_\theta (\theta sBd(A)) = \phi$.

(7) $\theta sBd(A) = A \setminus sInt_\theta (A)$.
$= A \setminus (X \setminus sCl_\theta (X \setminus A))$.
$= A \cap sCl_\theta (X \setminus A)$.

(8) $\theta sBd(A) = A \setminus sInt_\theta (A)$
$= A \setminus (A \setminus \theta sBd(X \setminus A))$.
$= \theta sBd(X \setminus A)$.

The proofs of (1), (2), (3), (4) and (6) are obvious.

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