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Research Paper

Solving Intuitionistic Fuzzy Assignment Problem by using Similarity Measures and Score Functions

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Abstract: *Classical Assignment Problem (AP) is a well-known topic world-wide. In this problem c_{ij} denotes the cost for assigning the j^{th} job to the i^{th} person. This cost is usually deterministic in nature. But in realistic situations, it may not be practicable to know the precise values of these costs. In such uncertain situations, instead of exact values of costs, if we can evaluate the preferences for assigning the j^{th} job to the i^{th} person in the form of composite relative degree (d_{ij}) of similarity to ideal solution (maximum degree indicates most preferable combination), we can replace c_{ij} by d_{ij} in the classical AP in the maximization form and can solve it by any standard procedure to get the optimal assignment. In this paper the cost c_{ij} has been considered to be intuitionistic fuzzy numbers (IFN) denoted by $\tilde{c}_{ij}$ which involves the positive and the negative evidence for the membership of an element in a set. It is a more realistic description than using the crisp and fuzzy concept. The similarity measures of intuitionistic fuzzy sets have been used in this paper for determining the composite relative degree of similarity d_{ij}. The notion of score function has also been used for validating the solution obtained by the composite relative similarity degree method. Numerical examples show the effectiveness of the proposed method for handling the Intuitionistic Fuzzy Assignment Problem (IFAP). Mathematical formulation of IFAP has been presented in this paper.*

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Keywords: Intuitionistic Fuzzy Assignment Problem;Intuitionistic Fuzzy sets, Intuitionistic Fuzzy Number, Similarity measures of Intuitionistic Fuzzy Sets, Score Function, Mathematical formulation.

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1. Introduction: In recent years, fuzzy transportation and fuzzy assignment problems have received much attention. Lin and Wen solved the assignment problem with costs in the form of fuzzy interval number by a labeling algorithm ([15]). In the paper by Sakawa et al [36], the authors dealt with actual problems on production and work force assignment in a housing material manufacturer and a subcontract firm and formulated two kinds of two-level programming problems. Chen [10] proved some theorems and proposed a fuzzy assignment model that considers all individuals to have same skills. Wang [44] solved a similar model by graph theory. Dubois and Fortemps [16] surveys refinements of the ordering of solutions supplied by the max–min formulation. Different kinds of fuzzy transportation problems are solved in the papers ([7], [8], [9], [34], [43]). Another Assignment problem with restrictions on time limits for jobs can be found in the numerical example of the papers ([23], [24]).
The concept of IFS can be viewed as an appropriate/alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In fuzzy sets the degree of acceptance is considered only but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. Presently intuitionistic fuzzy sets are being studied and used in different fields of science. Among the research works on these sets we can mention Atanassov ([1], [2], [3]), Szmidt and Kacprzyk ([38]-[42]), Buhaescu [4], Deschrijver and Kerre [18], Stoyanova [37]. With the best of our knowledge, Burillo et al.[5] proposed definition of intuitionistic fuzzy number and studied perturbations of intuitionistic fuzzy number and the first properties of the correlation between these numbers. Mitchell [32] considered the problem of ranking a set of intuitionistic fuzzy numbers to define a fuzzy rank and a characteristic vagueness factor for each intuitionistic fuzzy number. The intuitionistic fuzzy sets were first introduced by K. Atanassov [1] which is a generalization of the concept of fuzzy set [51]. The intuitionistic fuzzy set has received much attention since its appearance. Gau and Buehrer [19] introduced the concept of vague sets. But Bustince and Burillo [5] showed that vague sets are intuitionistic fuzzy sets and has been applied to many fields since

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its appearance. The theory of the IF set has been found to be more useful to deal with vagueness and uncertainty in decision situations than that of the fuzzy set ([1]-[3], [18], [38]-[42]). Over the last decades, the IF set theory has been successfully applied to solve decision making problems ([12], [20], [25]-[28], [38]-[42], [30], [35], [45]-[48]). Many authors have worked on the concept of similarity measures of fuzzy sets ([11], [24], [32]). Chen et al. (1995) [13] examined the similarity measures of fuzzy sets, which are based on the geometric model, set-theoretic approach, and matching function. Similarity measures of Intuitionistic fuzzy sets has been studied and modified by several authors. ([13], [52], [31], [50], [47]) . Chen (1988) [11] and Chen et al. (1995) [13] introduced a matching function to calculate the degree of similarity between fuzzy sets. Later on, this has been extended by Zeshui Xu, 2007 [47] to deal with the similarity measure of IFSs which has been successfully applied in solving a variety of Multi-Attribute Decision Making Problem ([47], [48]). Other applications of similarity measures of IFSs are in the fields, such as pattern recognition ([17], [29], [33]), description and classification of complex structured objects ([22], [6]) etc.

Assignment Problem (AP) is a well-known topic in Operations Research and is used worldwide for solving different types of problems in engineering and management science. Existing AP cost matrix contains deterministic and fixed values. But in a situation, when decision Maker (DM) has doubt to decide those costs, DM may induce the idea of acceptance and rejection bound on the costs. The costs may have a target value with degree of acceptance as well as degree of rejection. This fact seems to take the costs as an intuitionistic fuzzy set: a generalization of fuzzy set and is a more realistic description than the conception of crisp and fuzzy sense. These types of AP with intuitionistic fuzzy parameters are yet to be explored. In this paper, we developed a methodology for solving the Intuitionistic Fuzzy Assignment Problem (IFAP) by using similarity measures of IFSs. This new solution approach for IFAP cannot be found in the literature so far. We have validated our method with score function method through a numerical example.

We arrange the paper in the following way. In Section 2, we describe some preliminary ideas on IFSs and their similarity measures. In Section 3 we develop the mathematical model of the problem. In Section 4, we describe the solution procedure and in Section 5 we illustrate the method by suitable examples. Section 6 concludes the paper.

2. Preliminaries on Intuitionistic Fuzzy Sets

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2.1 Intuitionistic Fuzzy Set (IFS)

Let X= { $x_1, x_2, ..., x_n$} be a universe of discourse. A fuzzy set $A = \{< x_j, \mu_{\tilde{A}}(x_j) > / x_j \in X\}$, defined by Zadeh (1965) [51] is characterized by a membership function $\mu_{\tilde{A}} : X \rightarrow [0,1]$, where $\mu_{\tilde{A}}(x_j)$ denotes the degree of membership of the element x_j to the set A. Atanassov (1986) [1] introduced a generalized fuzzy set called IFS, shown as follows: An IFS A in X is an object having the form: $A = \{< x_j, \mu_{\tilde{A}}(x_j), \nu_{\tilde{A}}(x_j) > / x_j \in X\}$ which is characterized by a membership function $\mu_{\tilde{A}}$ and a non-membership function $\nu_{\tilde{A}}$, where $\mu_{\tilde{A}} : X \rightarrow [0,1]$, $x_j \in X \rightarrow \mu_{\tilde{A}}(x_j) \in [0,1]$, $\nu_{\tilde{A}} : X \rightarrow [0,1]$, $x_j \in X \rightarrow \nu_{\tilde{A}}(x_j) \in [0,1]$, with the condition $\mu_{\tilde{A}}(x_j) + \nu_{\tilde{A}}(x_j) \leq 1$, for all $x_j \in X$

For each IFS A in X, if $\pi_{\tilde{A}}(x_j) = 1 - \mu_{\tilde{A}}(x_j) - \nu_{\tilde{A}}(x_j)$, then $\pi_{\tilde{A}}(x_j)$ is called the degree of indeterminacy or hesitancy of x_j to A. Especially, if

$$\pi_{\tilde{A}}(x_j) = 1 - \mu_{\tilde{A}}(x_j) - \nu_{\tilde{A}}(x_j) = 0, \text{ for each } x_j \in X$$

then the IFS A is reduced to a fuzzy set.

2.1.1 Definition: Intuitionistic Fuzzy Number

An intuitionistic fuzzy number $\tilde{A}^i$ is defined as follows:

i) an intuitionistic fuzzy sub set of the real line

ii) normal i.e. there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = 1$ (so $\nu_{\tilde{A}^i}(x_0) = 0$)

iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$ i.e.

$$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2 \geq \min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \in R, \lambda \in [0,1]$$

iv) a concave set for the non-membership function $\nu_{\tilde{A}^i}(x)$ i.e.

$$\nu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2 \leq \max(\nu_{\tilde{A}^i}(x_1), \nu_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \in R, \lambda \in [0,1]$$

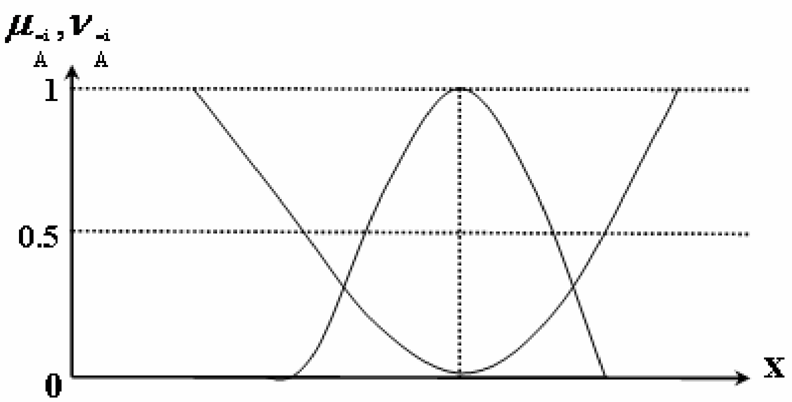

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Figure 1. Membership and non-membership functions of IFN

2.1.2 Ranking of Intuitionistic Fuzzy Numbers

Let $\tilde{a} = (\mu_1, \nu_1)$ be an intuitionistic fuzzy number.

Chen and Tan (1994) introduced a score function S of an intuitionistic fuzzy value, which is represented as follows.

Then $S(\tilde{a}) = \mu_1 - \nu_1$ be the score of $\tilde{a}$ where $S(\tilde{a}) \in [-1,1]$. The larger the score $S(\tilde{a})$, the greater the intuitionistic fuzzy value $\tilde{a}$.

2.2 Similarity Measures of Intuitionistic Fuzzy Sets

Let $\Phi(X)$ be the set of all IFSs of X.

Definition: Let s: $\Phi(X)^2 \to [0,1]$, then the degree of similarity between $A \in \Phi(X)$ and $B \in \Phi(X)$ is defined as $s(A,B)$, which satisfies the following properties:

1. $0 \le s(A,B) \le 1$;
2. $s(A,B) = 1$ iff $A = B$
3. $s(A,B) = s(B,A)$;
4. $s(A,C) \le s(A,B)$ and $s(A,C) \le s(B,C)$, if $A \subseteq B \subseteq C$, $C \in \Phi(X)$

2.2.1 Similarity measures based on matching function

Chen (1988) and Chen et al. (1995) introduced a matching function to calculate the degree of similarity between fuzzy sets. In the following, the matching function has been extended to deal with the similarity measure of IFSs.

Let $A \in \Phi(X)$ and $B \in \Phi(X)$, then the degree of similarity of A and B has been defined based on the matching function as:

$$s(A,B) = \frac{\sum_{j=1}^{n} (\mu_A(x_j).\mu_B(x_j) + \nu_A(x_j).\nu_B(x_j) + \pi_A(x_j).\pi_B(x_j))}{\max(\sum_{j=1}^{n} (\mu_A^2(x_j) + \nu_A^2(x_j) + \pi_A^2(x_j)), \sum_{j=1}^{n} (\mu_B^2(x_j) + \nu_B^2(x_j) + \pi_B^2(x_j)),} \quad (1)$$

Considering the weight w_j of each element $x_j \in X$, we get

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$$s(A,B)=\frac{\sum_{j=1}^{n} w_j(\mu_A(x_j).\mu_B(x_j)+\nu_A(x_j).\nu_B(x_j)+\pi_A(x_j).\pi_B(x_j))}{\max(\sum_{j=1}^{n} w_j(\mu_A^2(x_j)+\nu_A^2(x_j)+\pi_A^2(x_j)),\sum_{j=1}^{n} w_j(\mu_B^2(x_j)+\nu_B^2(x_j)+\pi_B^2(x_j)),} \quad (2)$$

If each element $x_j \in X$ has the same importance, then (2) is reduced to (1). The larger the value of $s(A,B)$, the more is the similarity between A and B. Here $s(A,B)$ has all the properties described as listed in the Definition in Section 2.2.

3. Mathematical Model:

Let there be n persons and n jobs. Each job must be done by exactly one person and one person can do, at most, one job. The problem is to assign the persons to the jobs so that the total cost of completing all jobs becomes minimum.

In this problem c_{ij} denotes the cost for assigning the j^{th} job to the i^{th} person.

We introduce the 0-1 variable x_{ij}, where

$$x_{ij}=\begin{cases}1, & \text{if the person i is assigned the job j;} \quad i, j = 1, 2, \ldots\ldots, n\\ 0, & \text{otherwise}\end{cases}$$

Corresponding to the (i j) th event of assigning person i to job j , the constraint

$\sum_{i=1}^{n} x_{ij}=1$, j = 1, 2,…., n means that each job must be done by exactly one person, and the constraint $\sum_{j=1}^{n} x_{ij}=1$, i = 1, 2, n means each person must be assigned at most one job.

Thus the model for crisp Assignment Problem is given by

MODEL 1:

$$\text{Min } z = \sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij} \quad (3)$$

Subject to

$$\sum_{i=1}^{n} x_{ij}=1, \quad j = 1, 2,\ldots., n \quad (4)$$

$$\sum_{j=1}^{n} x_{ij}=1, \quad i = 1,2,\ldots\ldots,n \quad (5)$$

$$x_{ij}= 0 \text{ or } 1, \quad i, j = 1, 2, \ldots\ldots, n \quad (6)$$

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This cost c_{ij} is usually deterministic in nature. But in real situations, it may not be practicable to know the precise values of these costs. In such an uncertain situation, instead of exact values of costs, if we know the preferences for assigning the j^{th} job to the i^{th} person in the form of composite relative degree (d_{ij}) of similarity to ideal solution (maximum degree indicates most preferable combination), we can replace c_{ij} by d_{ij} in the classical assignment problem in the maximization form and can solve it by any standard procedure(Hungarian method or by any software) to get the optimal assignment.

In that case the model for the preference AP becomes

MODEL2: Max z = $\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij} x_{ij}$ (7)

Subject to $\sum_{i=1}^{n} x_{ij} = 1,$ j = 1, 2,…., n (4)

$\sum_{j=1}^{n} x_{ij} = 1,$ i = 1,2,……,n (5)

$x_{ij} =$ 0 or 1, i, j = 1, 2, ……, n (6)

The cost c_{ij} may be in any form representing uncertain data, like interval number, triangular or trapezoidal fuzzy number or even intuitionistic fuzzy number. In this paper c_{ij} has been considered to be intuitionistic fuzzy numbers denoted by $\tilde{c}_{ij}$ which involves the positive and the negative evidence for the membership of an element in a set. The intuitionistic fuzzy set is a generalization of fuzzy set and is a more realistic description involving more uncertainty compared to the crisp and fuzzy concept. The cost of person i doing the job j is considered as an intuitionistic fuzzy number $\tilde{c}_{ij} = \{(\mu_{ij}, \nu_{ij})\}$, i, j = 1,2,3. Here μ_{ij} denotes the degree of acceptance and ν_{ij} denotes the degree of rejection of the cost of doing the j^{th} job by the i^{th} person. This problem is more realistic in the sense that instead of cost we have used its degree of acceptance and rejection.

The problem is to determine the composite relative similarity degree d_{ij} to ideal solution, denoting the preferences of the i^{th} person for doing the j^{th} job and vice versa with the costs c_{ij} given in the form of intuitionistic fuzzy number.

Alternatively, if we replace $\tilde{c}_{ij}$ by $\tilde{c}_{ij} = \{(\mu_{ij}, \nu_{ij})\}$ then the equation (3) becomes

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Min z = $\sum_{i=1}^{n}\sum_{j=1}^{n}\{(\mu_{ij},\nu_{ij})\}x_{ij}$ (8)

Our objective is to maximize acceptance degree μ_{ij} and to minimize the rejection degree ν_{ij}.

So the objective function (3b) can be written as Maximize $z_1 = \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_{ij}x_{ij}$ (9)

Minimize $z_2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\nu_{ij}x_{ij}$ (10)

Hence the IFAP becomes a multi-objective LPP in the form

MODEL 3: Maximize $z_1 = \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_{ij}x_{ij}$ (9)

Minimize $z_2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\nu_{ij}x_{ij}$ (10)

Subject to $(\mu_{ij}+\nu_{ij}-1)x_{ij} \le 0$ (11)

$\mu_{ij}x_{ij} \ge \nu_{ij}x_{ij}$ (12)

$\nu_{ij}x_{ij} \ge 0$ (13)

$\sum_{i=1}^{n}x_{ij} = 1,$ j = 1, 2,…., n (4)

$\sum_{j=1}^{n}x_{ij} = 1,$ i = 1,2,……,n (5)

$x_{ij} =$ 0 or 1, i, j = 1, 2, ……, n (6)

The above multi-objective model for IFAP can again be written as a single objective function LPP in the form

MODEL 4: Maximize $z = \sum_{i=1}^{n}\sum_{j=1}^{n}(\mu_{ij}-\nu_{ij})x_{ij}$ (14)

Subject to the conditions (4), (5), (6), (11), (12) and (13)

4. Solution Procedure

The cost matrix for the given assignment problem has been considered. But it cannot be solved by the traditional Hungarian method, since the elements of this matrix are in the form of Intuitionistic Fuzzy numbers. So, the concept of relative degree of similarity measures to the positive ideal solution of Intuitionistic Fuzzy Sets has been applied for solving this

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Intuitionistic Fuzzy Assignment Problem (IFAP) with the cost matrix as the given matrix. The Algorithm for describing this methodology has been stated.

This method can be used for solving assignment problems with any finite number of persons or jobs, the data for the selection alternatives being intuitionistic fuzzy sets.

In the following, we shall extend the method for solving intuitionistic fuzzy decision-making problem for solving Intuitionistic Fuzzy Assignment Problem (IFAP). The algorithm of the method is as follows:

For an Intuitionistic Fuzzy Assignment problem, let $A = \{A_1, A_2, A_3, ..., A_m\}$ be a set of alternatives for a row or column in the Assignment (cost) Matrix, and let C be an attribute (like cost or time or profit etc.) describing the selection alternative. Assume that the characteristics of the alternative A_i are represented by the IFS as:

$A_i = \{\langle C, \mu_{A_i}(C), \nu_{A_i}(C)\rangle / C$ being the attribute describing the selection alternative}, i=1,2,3,…….,m where $\mu_{A_i}(C)$ indicates the degree that the alternative A_i satisfies the attribute C, $\nu_{A_i}(C)$ indicates the degree that the alternative A_i does not satisfy the attribute C, and $\mu_{A_i}(C) \in [0,1]$, $\nu_{A_i}(C) \in [0,1]$, $\mu_{A_i}(C) + \nu_{A_i}(C) \le 1$

Algorithm 1:

Input: Cost matrix with the data being IFN.

Output: Profit matrix with data being the composite relative degree of similarity to the ideal solution, representing the preference or suitability to offer j^{th} job to the i^{th} person or that the i^{th} person is chosen for performing the j^{th} job and hence the optimal assignment.

At first the relative degree of similarity for the jobs with respect to each person are evaluated by applying the concept of similarity measures of IFSs for solving Intuitionistic Fuzzy Multi-Attribute Decision-Making (Xu, 2007). The data of the first column of the Assignment (cost) matrix are considered initially.

Step1: Let $\pi_{A_i}(C) = 1 - \mu_{A_i}(C) - \nu_{A_i}(C)$, for all i.=1,2,3,…..,m. Determine the positive-ideal and negative-ideal solution based on intuitionistic fuzzy numbers, defined as follows, respectively:

$$A^+ = \{\langle \mu_{A^+}(C), \nu_{A^+}(C)\rangle\} \quad (15)$$

and

$$A^- = \{\langle C, \mu_{A^-}(C), \nu_{A^-}(C)\rangle\} \quad (16)$$

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where $\mu_{A^+}(C) = \max_i \{ \mu_{A_i}(C) \}$, $\nu_{A^+}(C) = \min_i \{ \nu_{A_i}(C) \}$ (17)

$\mu_{A^-}(C) = \min_i \{ \mu_{A_i}(C) \}$, $\nu_{A^-}(C) = \max_i \{ \nu_{A_i}(C) \}$ (18)

Step2: Based on the Equation (2), the following similarity measures of IFSs have been defined. Calculate the degree of similarity of the positive ideal IFS A^+ and the alternative A_i, and the degree of similarity of the negative ideal IFS A^- and the alternative A_i, using the following equations respectively. The degree of similarity of each alternative A_i and the positive ideal IFS A^+ is defined as:

$$s(A^+, A_i) = \frac{\mu_{A^+}(C).\mu_{A_i}(C) + \nu_{A^+}(C).\nu_{A_i}(C) + \pi_{A^+}(C).\pi_{A_i}(C)}{\max\{(\mu^2_{A^+}(C) + \nu^2_{A^+}(C) + \pi^2_{A^+}(C)), (\mu^2_{A_i}(C) + \nu^2_{A_i}(C) + \pi^2_{A_i}(C))\}} \quad (19)$$

i = 1,2,….n; j = 1,2,3, …….,n

Similarly, degree of similarity of each alternative A_i and the negative ideal IFS A^- is defined as:

$$s(A^-, A_i) = \frac{\mu_{A^-}(C).\mu_{A_i}(C) + \nu_{A-}(C).\nu_{A_i}(C) + \pi_{A^-}(C).\pi_{A_i}(C)}{\max\{(\mu^2_{A^-}(C) + \nu^2_{A^-}(C) + \pi^2_{A^-}(C)), (\mu^2_{A_i}(C) + \nu^2_{A_i}(C) + \pi^2_{A_i}(C))\}} \quad (20)$$

i = 1,2,….n; j = 1,2,3, …….,n

Step 3: Based on (19) and (20) calculate the relative similarity measure d_i corresponding to the alternative A_i as:

$$d_i = \frac{s(A^+, A_i)}{s(A^+, A_i) + s(A^-, A_i)}, \; i = 1,2,3,......,n \quad (21)$$

Clearly, the bigger the value of d_i, the more similar is A_i to the positive ideal IFS A^+ and hence better is the alternative A_i.

Step 4: Repeat Step 1 to Step 3 for the rest of the columns of the cost matrix and find the relative similarity measure d_i corresponding to the alternative A_i for these columns i.e. for the jobs with respect to the persons.

Step 5: With these relative similarity measure d_i of the jobs with respect to the persons, form the matrix R_1 where $[R_1] = [\, p_{ij} \,]_{nxn}$, p_{ij} is the relative similarity measure representing how much the j^{th} person prefers the i^{th} job considering all the intuitionistic fuzzy attributes. We

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put ε>0, a very small number (degree of similarity) in the positions of the matrix R_1 to denote the situation that the j^{th} person cannot be assigned to the i^{th} job for these positions, if the data in the original problem considers that option.

Step 6: Now find the relative similarity measure d_j for the persons with respect to each job. Consider the data of the first row of the cost matrix. Repeat Step 1 to Step 3 for this row and also the rest of the rows of the cost matrix and find the relative similarity measure for these rows i.e. the relative similarity measure of the persons with respect to the jobs.

Step 7: With these relative similarity measure d_j of the persons with respect to the jobs, form the matrix R_2 where $[R_2] = [q_{ij}]_{nxn}$, q_{ij} is the relative similarity measure representing how much the i^{th} job is suitable for the j^{th} person considering all the intuitionistic fuzzy attributes. We put ε>0, a very small number (degree of similarity) in the positions of R_2 to denote the situation that the j^{th} person cannot be assigned to the i^{th} job for these positions, if the original problem considers this case.

Step 8: Then form the composite matrix Comp(R_1R_2) = $(p_{ij}\ q_{ij})_{nxn} = (d_{ij})_{nxn}$ whose elements are the composite relative degree of similarity representing the preference or suitability to offer the i^{th} job to the j^{th} person or that the j^{th} person is chosen for performing the i^{th} job.

Step 9: Then considering this matrix Comp (R_1R_2) as the initial table for an assignment problem in the maximization form (**MODEL4**), it is solved by Hungarian method or by any standard software to find the optimal assignment which maximizes the total composite relative degree of similarity.

Step 10: End.

Thus by using the above algorithm the more realistic Intuitionistic Fuzzy Assignment Problem (IFAP) can be solved.

4.1 Solution Procedure by using the Score Function of IF costs

Algorithm 2:

Input: Cost matrix with the data given in the form of IFN.

Output: Optimal Assignment.

Step 1: Find the Score function matrix of the given cost matrix with data in the form of IFN with the help of the formula defined in Section 5.1.2.

Step2: Considering this Score function matrix as the profit matrix in the maximization form, solve it by Hungarian Method or by any standard software to find the optimal assignment.

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5. Illustrative Examples

In this section, the application of the proposed approach has been demonstrated by evaluating the optimal assignment of projects to teams based on certain attributes which are represented by intuitionistic fuzzy numbers.

Example 1: Let us consider an Intuitionistic Fuzzy Assignment Problem (IFAP) having three machines and three jobs where the cost matrix contains intuitionistic fuzzy elements denoting time for completing the j^{th} job by the i^{th} machine. The cost matrix is given in Table 1. It is required to find the optimal assignment of jobs to machines.

Solution: This problem has been solved by using the Algorithm 1, since the data of the cost matrix are IFNs.

The results are shown in Table 2, Table 3 and Table 4.

Table1: Data for the Intuitionistic Fuzzy Assignment Problem (IFAP) without restrictions

Jobs / Machine	J_1	J_2	J_3
M_1	(0.4,0.5)	(0.6,0.2)	(0.5,0.2)
M_2	(0.2,0.8)	(0.8,0.1)	(0.6,0.4)
M_3	(0.7,0.3)	(0.3,0.6)	(0.4,0.3)

With the Table 4 as the profit matrix of the assignment problem in the maximization form (since our objective is to maximize the composite relative degree of similarity to the PIIFS), it has been solved by Hungarian method or by using any standard software. The optimal assignment is

1st Job is assigned to the 3rd Machine

2nd Job is assigned to the 2nd Machine and

3rd Job is assigned to the 1st Machine.

The above problem has also been solved by applying Algorithm 2 or by writing it in the form of the MODEL 4 and hence solving it. In all these cases the optimal assignment was found to be the same. The score function matrix for applying Algorithm 2 is shown in Table 5.

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Table 2: Values of $s(A^{+},M_j)$, $s(A^{-},M_j)$ and values of d_j in R_1 for Machines with respect to the Jobs (column wise)

Values of $s(A^{+},M_j)$ for Machines with respect to the Jobs (column wise)			
$s(A^{+},M_j)$	J_1	J_2	J_3
M_1	0.741	0.788	0.909
M_2	0.559	1.000	0.846
M_3	1.000	0.470	0.818
Values of $s(A^{-},M_j)$ for Persons with respect to the Jobs (column wise)			
$s(A^{-},M_j)$	J_1	J_2	J_3
M_1	0.706	0.696	0.895
M_2	1.000	0.470	0.769
M_3	0.559	1.000	0.944
Matrix R_1 containing the values of d_j for Machines with respect to the Jobs (column wise)			
d_j or p_{ij}	J_1	J_2	J_3
M_1	0.512	0.531	0.504
M_2	0.358	0.680	0.524
M_3	0.642	0.320	0.464

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Table 3: Values of $s(A^+, J_i)$, $s(A^-, J_i)$, d_i in the matrix R_2 for Jobs with respect to the Machines (row wise)

Values of $s(A^+, J_i)$ for Jobs with respect to the Machines (row wise)			
$s(A^+, J_i)$	J_1	J_2	J_3
M_1	0.818	1.000	0.909
M_2	0.353	1.000	0.788
M_3	1.000	0.672	0.638
Values of $s(A^-, J_i)$ for Jobs with respect to the Machines (row wise)			
$s(A^-, J_i)$	J_1	J_2	J_3
M_1	1.000	0.818	0.786
M_2	1.000	0.353	0.647
M_3	0.672	1.000	0.717
Matrix R_2 containing the values of d_i for Jobs with respect to the Machines (row wise)			
d_i or q_{ij}	J_1	J_2	J_3
M_1	0.450	0.550	0.536
M_2	0.261	0.739	0.549
M_3	0.598	0.402	0.445

Table 4: Composite Matrix representing the Composite relative degree of similarity (preferences) of Jobs and Machines

$Comp(R_1R_2) = (p_{ij}q_{ij})_{nxn} = (d_{ij})_{nxn} =$

$p_{ij}q_{ij}$	J_1	J_2	J_3
M_1	0.230	0.292	0.270
M_2	0.093	0.503	0.288
M_3	0.384	0.129	0.206

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Table 5: Scores of IF numbers of Table 5.1

S	J_1	J_2	J_3
M_1	-0.1	0.4	0.3
M_2	-0.6	0.7	0.2
M_3	0.4	-0.3	0.1

6. Conclusions

In this paper, the Intuitionistic Fuzzy Assignment Problems (IFAP) and its solution procedure has been introduced. The problem has been formulated and depicted by various mathematical models. The procedure for solving it has been described which uses the concept of relative degree of similarity to the PIIFS under IF environment. Moreover, here all the parameters are considered as IFNs which makes the problem more general and realistic in nature in the sense that it considers both the degree of acceptance and the degree of rejection. This method can be used for solving assignment problems with any finite number of persons or jobs, the data for the selection alternatives being IFN. Illustrative example shows practicality, effectiveness and importance of the developed approach. Since, no other algorithms are available at present to solve IFAP, the results obtained by applying the Algorithm5.2.1 are compared with that obtained by applying Algorithm 5.2.2 or by solving MODEL 5.2.4. The result obtained by the proposed method is validated with the same result obtained by solving the IFAP considering the score function matrix as the profit matrix. The numerical examples show that for a particular problem, all the methods result in similar optimal assignments. The results reveal that the proposed methodologies can effectively solve the IFAP. A type of similarity measures of IFSs has been used in this paper. Other types of similarity measures may be constructed and used for solving IFAP.

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